Measuring Market Risk with the Folded Peaks-Over-Thresholds Approach

Peter Julian Cayton
University of the Philippines Diliman

Abstract

This paper discusses the folding procedure for the peaks-over-thresholds (POT) models and their applications in market risk measurement, namely the value-at-risk (VaR) and the expected shortfall (ES). Folding is defined as a procedure in which when data fall below a certain threshold value, a transformation formula will move the data points above the threshold. First, an initial fitting with the generalized Pareto distribution (GPD) over a temporary threshold is done. Second, from the initially-fitted GPD estimates and a newly-selected threshold, a folding transformation moves the data points lower to the new threshold of higher values. Third, the data points higher than the new threshold are fit to the GPD for inference and risk estimation. The risk measures from the folded GPD approach are compared with the ARMA-GARCH financial econometric and the unfolded POT approach in terms of their performance in real financial time series data such as the stock indices and foreign currencies. The benefit of folding in the POT is that it lowers estimates of standard errors for the GPD parameters given that an appropriate threshold has been selected. These would indicate more accurate GPD parameter estimates that lead to better VaR and ES estimates. The real data application results show that the VaR and ES from the folded POT methodology have less exceedances. Loss calculations indicate that those folded POT might mean higher capital adequacy – the conservatively set VaR and ES would cushion from extreme losses incurred from exceedance events.

Keywords: *Value-at-Risk, Extreme Value Theory, Financial Risk Management*

1. Introduction

Financial institutions bear risks in their interaction with financial markets because of the uncertainty in the levels of financial returns in these activities. This kind of risk borne from transacting with the financial markets is defined as market risk. Financial institutions engage in activities in the financial markets, so they can accumulate assets that they may use to provide services to the public. However, the quality of service and the lifetime of financial institutions depend on the returns of financial assets, of which sudden losses in value of their investments might lead to closure. Thus regulators of financial institutions, such as the *Bangko Sentral ng Pilipinas* (BSP) for the Philippines, are setting up standards for capital adequacy with respect to the risk that these institutions are exposed. One of the approaches that BSP opens to institutions are the internal models approach, in which banks create risk models within their institutions that measure their levels of exposure, provided that these models meet standards of viability in forecasting risk [5].

Because of this, methodologies for estimating risk have flourished. Regulators propose the use of VaR [21] in the estimation of their risk capital. However, due to noncoherence of the value-at-risk, a coherent function of it, called ES [1], has been used as a risk measure. To compute both VaR and ES, thorough understanding of statistical properties of financial time series is necessary, so called stylized facts [24, 27]. Financial time series are non-stationary, thus the returns in holding an asset are used. The returns have nonnormal features of thick tails and skewness. Variance of returns have been confirmed to be conditionally dynamic in time.

A family of models that target thick tails are POT models of extreme value statistics which uses the GPD [4, 10, 13, 25]. The procedure involves only using returns data that have loss exceeding a threshold quantity. They have been used in modeling financial time series aimed in risk measurement [10, 24, 26, 27]. However, the problem with POT modeling is the data deletion of any value not filtered through the threshold. This introduces a high data requirement for using POT and the loss of information that can improve risk estimation. However, a remedy has been introduced in which unfiltered data through the threshold are folded above the threshold to be used for estimation of parameters and tail quantiles [18]. From this development, the paper devises a means of estimating VaR and ES based on the folded POT model.

The discussion flow of the paper is outlined as follows: The second section of the paper discusses the background literature on financial time series, mean-variance modeling, the POT approach, and the folding methodology. It also includes definitions of risk measures and the methodology of risk performance evaluation using exceedance tests and loss functions. The third section discusses the devised methodology of risk modeling with folding and mean-variance specifications in POT approach. Real data applications are also discussed, in which the methodology is compared with two other models and are fit into five financial time series. Three stock indices and two currency rates are used as real data. Results of application to real data are discussed in the fourth section. Summary statistics, model estimates, and risk performance are outlined. Finally, the paper is summarized and concludes the viability of the proposed model in the fifth section.

2. Background Literature

2.1. Stylized Facts of Financial Time Series

In the statistical analysis of financial time series data, we transform the price series of a financial asset to its returns [27]. For example, if the price of a zero-dividend financial instrument at time t is denoted by P_t for $t = 1, 2, \ldots, T$, then the percentage-change return of holding a financial instrument from $t_{initial}$ to $t_{current}$ is equal to

$$r_{t_{current},pchange} = \frac{P_{t_{current}} - P_{t_{initial}}}{P_{t_{initial}}} \times 100\% \quad (1)$$

Often, return series are computed between adjacent periods, so the formula simplifies to:

$$r_{t,pchange} = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100\% \quad (2)$$

Another formula for returns often used in statistical modeling of financial time series would be the difference of natural logarithms of adjacent prices, or log-returns:

$$r_t = [\log P_t - \log P_{t-1}] \times 100\% \quad (3)$$

This formula is used in this paper for statistical modeling because it restricts the prices of financial instruments to positive values.

Financial time series data do not exhibit the properties that are commonly assumed in statistical modelling and inference. The properties are: (1) non-normality of distribution of returns; and (2) volatility clustering or time-varying variance [24, 27].

Non-normality of returns may be broken down to two characteristics [16, 23]: (1) financial returns distributions tend to have thicker tails than the normal distribution, which means higher or positively infinite kurtosis, describing returns as leptokurtic; and (2) negative skewness or skewed in which tails are longer in the side of negative values. Having thicker tails than the normal distribution implies that the probabilities of observing values in the tails are higher than what the normal distribution can describe. Negative skewness in combination with thick tails means that for financial returns, the very large losses are more frequently observed than what would be expected from values generated based on the normal distribution. So, to assume normality when modeling financial returns would be invalid and would not be able to account the observed frequency of large losses.

Volatility clustering is the phenomenon in which large fluctuations in returns values in the immediate past values are followed by large fluctuations in more recent periods and small fluctuations are followed by small fluctuations. The changing sizes of fluctuations in time mean that variance conditionally changes based on the information available from the immediate past values of the current period. This kind of behavior is modeled by the autoregressive conditional heteroscedasticity (ARCH) specifications by Engle [15] and is extended by Bollerslev [7] through the Generalized ARCH (GARCH) models and generalized further of increasing complexity.

2.2. ARMA-GARCH Specification

The GARCH family of models [7] describe the property of conditional variances of time series data. To start, let r_t log-returns series as defined by equation 3 for time $t = 1, 2, \ldots, T$. The GARCH(p,q) equation specification is defined as:

$$r_t = \mu_t + u_t \tag{4}$$

$$u_t = \sqrt{h_t}\epsilon_t, \quad \epsilon_t \sim WhiteNoise(0,1) \tag{5}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{6}$$

Equation 4 describes the data-generating process of r_t with the term $\mu_t = E[r_t | I_{t-1}]$ as the mean specification, which the expected value of returns given the I_{t-1} = information available at time $t - 1$. Common mean specifications may be time series regression models [14] or Box-Jenkins autoregressive moving average (ARMA) [8] models; the second model is used for the paper. As a brief discussion of the Box-Jenkins ARMA (p', q') model, the equation is presented below:

$$r_t = \omega + \sum_{i=1}^{p'} \phi_i r_{t-i} + \sum_{j=1}^{q'} \theta_j u_{t-j} + u_t \tag{7}$$

The first summation set describes the autoregressive component, which is the dependence of the current value of r_t to its immediate past values r_{t-i} as weighted by the corresponding parameter φ_i. The second summation set describes the moving average components, which is the dependence of current value of r_t with past forecasting errors u_{t-j} with weight described by parameter θ_j. The intercept of the model is the parameter ω. Possible values of p' and q' are any nonnegative integers. Box and Jenkins [8] provides system of model fitting procedures to follow to get an appropriate fitting ARMA model. For this paper, the *auto.arima* procedure in R [20] is implemented which selected the orders of the ARMA(p', q') with the smallest Akaike information criterion (AIC) [2] value:

$$AIC = -2 \log L + 2k \tag{8}$$

where $k = p' + q' + 1$ if an intercept ω is included and $k = p' + q'$ if otherwise.

The interaction of the conditional variance $h_t = var[r_t | I_{t-1}]$ and the white noise process ϵ_t is described in equation 5. A common error distribution for GARCH estimation is the normal distribution with a correction on the standard errors of the estimators to account for heteroskedasticity, called quasi-maximum likeli- hood estimation [28]. This does not drastically alter the estimated parameters, but it does affect significance testing for the parameters.

Equation 6 is the GARCH equation [7], which shows the dependence of the conditional variance h_t to immediate past squared raw errors u^2_{t-i} with the speed of adjustment to past errors described by α_i and to immediate past variance values h_{t-j} with weights described by parameters β_j. This term facilitates time-varying variance and the observed occurrence of volatility clustering. The quick increases in fluctuations are emulated by large values of α_i, while the lingering duration of variances remaining high are emulated by large values of β_j. The possible values of the model with orders p and q are any nonnegative integer. The appropriate GARCH(p,q) order may be selected by iterative use of formal statistical tests such as the Ljung-Box white noise test [22] on squared residuals evaluated on sufficiently large number of lags.

2.3. Peaks-Over-Thresholds (POT) Approach

With respect to thicker tails in return distributions and the focus on losses of holding assets in financial activities, a common technique in risk measurement and estimation is to make inferences and forecasts based on modeling tail behavior. To describe the tails, a threshold value should be described first which segregates the tail values appropriate for fitting and the rest of the data discarded from the statistical modeling approach; thus it is called POT approach in extreme value theory (EVT) [10, 13].

A distribution to fit for the POT approach is the GPD [4, 25]. The notation for a random variable X following the GPD distribution is $X \sim GPD(\delta, \sigma, \xi)$ and the density fx, the distribution F_x, the quantile function F_X^{-1}, and the mean of the distribution $E(X)$ are described below [10, 13]:

$$f_X(x;\delta,\sigma,\xi) = 1 - \left(1+\xi\frac{x-\delta}{\sigma}\right)^{-\frac{1}{\xi}}, \qquad x \geq \delta, \quad 1+\xi\frac{x-\delta}{\sigma} \geq 0 \qquad (9)$$

$$F_X(x;\delta,\sigma,\xi) = \frac{1}{\sigma}\left(1+\xi\frac{x-\delta}{\sigma}\right)^{-\frac{1}{\xi}-1}, \qquad x \geq \delta, \quad 1+\xi\frac{x-\delta}{\sigma} \geq 0 \qquad (10)$$

$$F_X^{-1}(u;\delta,\sigma,\xi) = \delta + \sigma\frac{u^{-\xi}-1}{\xi}, \qquad 0 \leq u \leq 1 \qquad (11)$$

$$E(X|\delta,\sigma,\xi) = \delta + \frac{\sigma}{1-\xi}, \qquad if \quad \xi < 1 \qquad (12)$$

Note that the above equations assume a right-side tail. For the returns distribution of financial assets, a negative transformation can be done in data preprocessing for density fitting. Negative transformation of returns may be called the loss distribution of holding the financial asset.

The scale parameter of the GPD is σ. Different values of the parameter, holding other parameters fixed, simply stretch or compress the density function without affecting its general shape or location. The parameter ξ is the shape parameter, which can dictate whether the distribution will have a finite or infinite right endpoint. If $\xi \geq 0$, then $x \geq 0$ for equations 9 and 10. It also describes whether the mean of the GPD in equation 12 is finite when $\xi < 1$ or infinite otherwise.

The parameter δ is the threshold parameter. This is not directly estimated by any statistical method using the data when using the POT approach. This is selected by the user through threshold selection techniques. In this paper, the Hill estimator plot [13, 19] is used for the selection of the appropriate threshold. The Hill estimator for the GPD provides an initial estimate on the shape parameter ξ. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $GPD(\delta, \sigma, \xi)$. Let $X_{(i)}$ = the ith smallest value in the random sample. The Hill estimator $\hat{\xi}_{Hill,k}$ is:

$$\hat{\xi}_{Hill,k} = \left(\frac{1}{k} \sum_{i=0}^{k-1} \log\left(X_{(n-i)}\right) - \log\left(X_{(n-k)}\right) \right)^{-1} \tag{13}$$

With the choice of $k = k(n)$ as a function of n with $k(n) \rightarrow 0$, the Hill estimator $\hat{\xi}_{Hill,k} \longrightarrow \xi$ as sample size $n \longrightarrow \infty$. The Hill estimator plot is a 2-dimensional plot such that the different choices for k are on the x-axis while the different values of $\hat{\xi}_{Hill,k}$ are on the y-axis. This implies that an appropriate threshold would be $\hat{\delta} = X_{(n-k^*)}$ such that k^* is small and the estimated $\hat{\xi}_{Hill,k}$ value will be a constant from $k = k^*$ to $k = 0$, or the edge of the plot.

When an appropriate threshold has been selected, estimation of the GPD parameters σ and ξ may be done. This paper used maximum likelihood estimation (MLE) [10, 13]. Let $(X_{(n-k^*+1)}, X_{(n-k^*+2)}, \ldots, X_{(n)})$ be the set of data from the original data $(X_1, X_2, \ldots, X_n)$ of sample size n with which $X_i > X_{(n-k*)}$ and let $\hat{\delta} = X_{(n-k^*)}$. The maximum likelihood estimators of (σ, ξ) are:

$$(\hat{\sigma}_{ML,\hat{\delta}}, \hat{\xi}_{ML,\hat{\delta}}) = \underset{\sigma>0,\xi\in\mathbb{R}}{\arg\max} \sum_{i=0}^{k-1} \log[f_X(x_{(n-i)}; \hat{\delta}, \sigma, \xi)] \tag{14}$$

Thus, the POT procedure is as follows:

Step 1: Select the approach threshold $\hat{\delta} = X_{(n-k^*)}$ using the Hill plot and equation 13.

Step 2: In the sample $(X_1, X_2, \ldots, X_n)$, extract $(X_{(n-k^*+1)}, X_{(n-k^*+2)}, \ldots, X_{(n)})$ and get the maximum likelihood estimates of (σ, ξ) using the set-up of equation 14

Step 3: From the results of step 2, one can perform inferences or estimations of extreme quantiles. Note the GPD quantiles are different from POT by a correction based on the deletion of data in step 1:

$$\hat{F}^{-1}_{X,POT}(u; \hat{\delta}, \hat{\sigma}, \hat{\xi}) = \hat{\delta} + \hat{\sigma} \frac{\left(\frac{n}{k^*} u\right)^{-\hat{\xi}} - 1}{\hat{\xi}}, \quad 0 \leq u \leq 1 \tag{15}$$

2.4. Folding in Extreme Value Statistics

A problem with POT approach is it discards data until a sample size k^* remains even after having a large initial sample size n. This means that a lot of information that can be extracted from the initial sample is discarded and that a large sample size n is necessary to assure that a large truncated sample k^* would remain after filtering through a threshold $X_{(n-k^*)}$. Also, since k^* is a smaller sample size, it may result to higher standard errors of ML estimates, since ML estimates of basic parameters of distributions tend to be negative power functions of sample size [9].

However, a means of remedy on the problem of sample size is folding methodology in EVT [18], based on perfect sampling techniques used in simulation studies [11] and the idea of connecting the two tails of a distribution.

The folding methodology is outlined below:

Step 1: Select a preliminary threshold $\tilde{\delta} = X_{(n-k_F)}$ and estimate (σ, ξ) using $(\tilde{\sigma}_{ML,\tilde{\delta}}, \tilde{\xi}_{ML,\tilde{\delta}})$ from

$$(\tilde{\sigma}_{ML,\tilde{\delta}}, \tilde{\xi}_{ML,\tilde{\delta}}) = \underset{\sigma>0,\xi\in\mathbb{R}}{\arg\max} \sum_{i=0}^{k_F-1} \log[f_X(x_{(n-i)}; \tilde{\delta}, \sigma, \xi)] \tag{16}$$

Step 2: Select a second threshold $\hat{\delta} = X_{(n-k^*)}$ where $\hat{\delta} > \tilde{\delta}$ and calculate $\tilde{\sigma}_{F,\tilde{\delta}} = \tilde{\sigma}_{ML,\tilde{\delta}} + \tilde{\xi}_{ML,\delta}(\hat{\delta} - \tilde{\delta})$

Step 3: Perform the folding transformation on all the order statistics of the original sample data of size *n* as shown below with the use of equation 11:

$$X_{(i),F} = \begin{cases} F_X^{-1}(u = 1 - \frac{i}{k}; \delta = \hat{\delta}, \sigma = \tilde{\sigma}_{F,\tilde{\delta}}, \xi = \tilde{\xi}_{ML,\tilde{\delta}}) & \text{if } i < k^* \\ X_{(i)} & \text{if } i \geq k^* \end{cases} \tag{17}$$

Step 4: From the folded sample $(X_{(1),F}, X_{(2),F}, ..., X_{(n),F})$, estimate the GPD parameters (σ, ξ) using $(\hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\mu}})$ using

$$(\hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\delta}}) = \underset{\sigma>0,\xi\in\mathbb{R}}{\arg\max} \sum_{i=0} \log[f_X(x_{(n-i)}; \hat{\delta}, \sigma, \xi)] \tag{18}$$

Step 5: Estimate the POT extreme quantiles with correction based on data deletion:

$$\hat{F}^{-1}_{X,FPOT}(u; \hat{\delta}, \hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\delta}}) = \hat{\delta} + \hat{\sigma}_{FML,\hat{\delta}} \frac{\left(\frac{n}{k^*}u\right)^{-\hat{\xi}_{FML,\hat{\delta}}} - 1}{\hat{\xi}_{FML,\hat{\delta}}}, \quad 0 \leq u \leq 1 \tag{19}$$

Based on the simulation studies [18], the quantile $\hat{F}^{-1}_{X,F,POT}$ has lower bias and root mean-square error in estimating the true extreme quantiles on scenarios involving the Burr and the standard Frechet distribution, which are heavy- tailed distributions. It also showed that the ML estimators of equation 18 are asymptotically normal. These properties on extreme quantiles are vital in estimating a financial risk using the POT approach.

2.5. Risk Measurement

Since market risks are incurred by financial institutions in their transactions with different asset markets, these activities are placed in check by regulators. Institutions keep track of the risks that they incurred by measuring it. There are two risk formulas used in monitoring: (1) VaR [21] and (2) (ES) [1].

Value-at-risk $VaR_{t,1-\alpha}$ at time t and probability $1-\alpha$ is the minimum loss of holding an asset, given a probability α that such loss may be exceeded [21]. In terms of statistical methodology, this is the upper α quantile of a loss distribution. In equation form, with $L_t = -r_t$ as the loss at time t

$$VaR_{t,1-\alpha} : P\left[L_t > VaR_{t,1-\alpha}\right] = \alpha. \tag{20}$$

Given that the VaR is exceeded, the expected loss is called the expected shortfall [1]. This is achieved by solving the mean at the set of values of L_t where $L_t > VaR_{t,1-\alpha}$. The definition formula for ES is:

$$ES_{t,1-\alpha} = E\left[L_t | L_t > VaR_{t,1-\alpha}\right] \tag{21}$$

ES risk formulas are said to comply [1] with the definition of coherent risk measures [3], which are: (1) monotonous, which means a risk measure should be always negative if computed using the return distribution, (2) sub-additive, which means the risk of a portfolio, computed using the return distribution, should be less than the sum of the risk of its components, (3) positively homogeneous, which is if one buys more of an asset by a multiplier $h > 0$, the risk also increases by a factor of h, and (4) translation equivariant, which is when a constant a is added to a portfolio, the risk accounts for the addition. VaR is not sub-additive, thus it is not coherent. ES is coherent as it has been recommended over VaR.

Given special distributions, the formula for the VaR and ES are simplified. For the loss L_t having a normal distribution with forecasted mean $\hat{\mu}_t$ and forecasted conditional variance $\hat{h}_t$, the standard normal distribution function $\Phi(z)$, and standard normal density $\varphi(z)$, the VaR and ES formulas will be [24]:

$$VaR^{Normal}_{t,1-\alpha} = \hat{\mu}_t + \sqrt{\hat{h}_t}\Phi^{-1}(1-\alpha) \tag{22}$$

$$ES^{Normal}_{t,1-\alpha} = \hat{\mu}_t + \sqrt{\hat{h}_t}\frac{\phi\left[\Phi^{-1}(1-\alpha)\right]}{\alpha} \tag{23}$$

Under the POT model for the loss distribution, the VaR, based on equation 15, and ES are shown below:

$$VaR^{POT}_{1-\alpha} = \hat{F}^{-1}_{X,POT}(1-\alpha; \hat{\delta}, \hat{\sigma}_{ML,\hat{\delta}}, \hat{\xi}_{ML,\hat{\delta}}) \tag{24}$$

$$ES^{POT}_{1-\alpha} = \frac{VaR^{POT}_{1-\alpha}}{1-\hat{\xi}_{ML,\hat{\delta}}} + \frac{\hat{\sigma}_{ML,\hat{\delta}} - \hat{\xi}_{ML,\hat{\delta}}\hat{\delta}}{1-\hat{\xi}_{ML,\hat{\delta}}} \tag{25}$$

As the POT models are static in the time series sense without adapting to the changing conditions of time series data, Suaiso and Mapa [26] proposed an ARMA(p,q)-GARCH(p', q')-POT model. The Suaiso-Mapa methodology involves a two-step procedure:

Step 1: Fit an ARMA(p,q)-GARCH(p', q') model on the return series r_t and extract the standardized residuals e_t of the model.

Step 2: Perform the peak-over-thresholds procedure on the standardized residuals e_t and extract the estimated parameters of the GPD $(\hat{\delta}, \hat{\sigma}_{ML,\hat{\delta}}, \hat{\xi}_{ML,\hat{\delta}})$

The VaR and ES for the Suaiso-Mapa method for the loss distribution is shown below:

$$VaR_{t,1-\alpha}^{AGPOT} = \mu_t + \sqrt{\hat{h}_t} \hat{F}_{X,POT}^{-1}(1-\alpha; \hat{\delta}, \hat{\sigma}_{ML,\hat{\delta}}, \hat{\xi}_{ML,\hat{\delta}}) \tag{26}$$

$$ES_{t,1-\alpha}^{AGPOT} = \mu_t + \sqrt{\hat{h}_t} \left[\frac{\hat{F}_{X,POT}^{-1}(1-\alpha; \hat{\delta}, \hat{\sigma}_{ML,\hat{\delta}}, \hat{\xi}_{ML,\hat{\delta}})}{1-\hat{\xi}_{ML,\hat{\delta}}} + \frac{\hat{\sigma}_{ML,\hat{\delta}} - \hat{\xi}_{ML,\hat{\delta}}\hat{\delta}}{1-\hat{\xi}_{ML,\hat{\delta}}} \right] \tag{27}$$

The Suaiso-Mapa methodology has been assessed to be more dynamic with smaller frequency of VaR exceedances. The proposed procedure is based on the Suaiso-Mapa method. The two methodologies and GARCH-Normal VaRs with their corresponding ES formulas will be assessed and compared in terms of their performance using evaluation approaches to risk measures.

2.6. Evaluation of Risk Measurements

Because of the myriad of risk estimation methodologies that exist in literature, the choice of the optimal risk methodology for a specific problem is pursued and many criteria are devised.

The first of the criteria are the Christoffersen Likelihood Tests [12], which are the tests for unconditional coverage LR_{uc}, independence LR_{ind}, and conditional coverage LR_{cc}.

Unconditional coverage test examines whether the $VaR_{t,1-\alpha}$ model fails at the appropriate proportion α of exceedances; e.g., for the $VaR_{t,0.99}$, it may only fail at most one percent of the time. Suppose that T_1 is the number of exceedances from $VaR_{t,1-\alpha}$ model. The VaR failure rate p is estimated by $\hat{p} = \frac{T_1}{T}$ where T is the length of time for evaluating the VaR model. The LR_{uc} test statistic for $H_0 : p = \alpha$ versus the alternative $H_A : p \neq \alpha$ is:

$$LR_{uc} = 2\log\left[\frac{(1-\hat{p})^{T-T_1}(\hat{p})^{T_1}}{(1-\alpha)^{T-T_1}(\alpha)^{T_1}}\right] \tag{28}$$

The null hypothesis is rejected at α_{sig} level of significance if $LR_{uc} > \chi^2_{\alpha_{sig}, df=1}$. It should be noted that it is possible to reject H_0 even if $\hat{p} < \alpha$ which means the VaR model performs better than expected.

The test for independence examines whether or not the exceedances in the previous period would be followed by another exceedance. A preferred property is that current exceedance should be independent of past exceedances. Still using the quantities defined in the unconditional coverage test, additional inputs are necessary.

Two parameters are assumed by this test: p_0 is the parameter that describes the probability of VaR exceedance at the current period given that the previous period did not have VaR exceedances, and p_1 is the probability of VaR exceedance at the current period given a previous period with VaR exceedance. Let T_{10} be the number of instances that exceedances are followed by non-exceedances, T_{11} be the number of instances of two consecutive exceedances, T_{01} is the number of non-exceedances followed by exceedances, and T_{00} is the number of instances of two nonexceedances.

From these quantities, $\hat{p}_0 = \frac{T_{01}}{T_{00}+T_{01}}$ and $\hat{p}_1 = \frac{T_{11}}{T_{10}+T_{11}}$. The LR_{ind} test statistics for $H_0 : p_0 = p_1$, independent exceedances, versus the alternative $H_A : p_0 \neq p_1$ serially correlated exceedances, is

$$LR_{ind} = 2\log\left[\frac{(1-\hat{p}_0)^{T_{00}}(\hat{p}_0)^{T_{01}}(\hat{p}_1)^{T_{11}}(1-\hat{p}_1)^{T_{10}}}{(1-\hat{p})^{T-T_1}(\hat{p})^{T_1}}\right] \tag{29}$$

The null hypothesis is rejected at α_{sig} level of significance if $LR_{ind} > \chi^2_{\alpha_{sig},df=1}$. The test for conditional coverage is the test in which independent exceedances still comply with desired coverage probabilities. The LR_{cc} test statistic is simple, as it is the sum of the two previous tests:

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{30}$$

The null hypothesis is rejected at α_{sig} level of significance if $LR_{cc} > \chi^2_{\alpha_{sig},df=2}$.

The other risk evaluation that was used is the VaR-based loss function $\tilde{Q}$[17]. For each time point t, the value of $\tilde{Q}(t)$ for a $VaR_{t,1-\alpha}$ model derived from the loss function is:

$$\tilde{Q}(t) = \left(\alpha - [1+\exp\{25[r_t - (-VaR_{t,1-\alpha})]\}]^{-1}\right)[r_t - (-VaR_{t,1-\alpha})] \tag{31}$$

The VaR-based loss function is solved on the evaluation periods of the VaR model under consideration. It means that the data will have to be split into two parts: (1) estimation or training periods for generating the parameters of the VaR model, and (2) evaluation or testing periods for forecasting VaR but were not included in the estimation of the VaR model parameters.

In analyzing the loss function, statistics of interest are the average loss and the maximum loss values. The best case would be to observe lower loss values and lower loss statistics.

With the different methodologies of comparing VaR model performance discussed, the methodology of the paper is discussed in the next section.

3. Methodology

3.1. VaR and ES using GARCH-Folded EVT Procedure

In the same line of reasoning as [26], a dynamic ARMA(p,q)-GARCH(p/, q/)-Folded POT model is devised by the paper. The proposed methodology involves a two-step procedure:

Step 1: Fit an ARMA(p,q)-GARCH(p', q') model on the return series r_t and extract the standardized residuals e_t of the model.

Step 2: Perform the folded POT procedure on the standardized residuals et and extract the estimated parameters of the GPD, $(\hat{\delta}, \hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\delta}})$

The VaR and ES of the proposed procedure for the loss distribution is shown below:

$$VaR_{t,1-\alpha}^{AGFPOT} = \mu_t + \sqrt{\hat{h}_t}\hat{F}_{X,POT}^{-1}(1-\alpha; \hat{\delta}, \hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\delta}}) \quad (32)$$

$$ES_{t,1-\alpha}^{AGFPOT} = \mu_t + \sqrt{\hat{h}_t}\left[\frac{\hat{F}_{X,POT}^{-1}(1-\alpha; \hat{\delta}, \hat{\sigma}_{FML,\hat{\delta}}, \hat{\xi}_{FML,\hat{\delta}})}{1-\hat{\xi}_{FML,\hat{\delta}}} + \frac{\hat{\sigma}_{FML,\hat{\delta}} - \hat{\xi}_{FML,\hat{\delta}}\hat{\delta}}{1-\hat{\xi}_{FML,\hat{\delta}}}\right] \quad (33)$$

The performance of the VaR equations 22, 26, and 32 and ES equations 23, 27, and 33 are assessed on real data with evaluation measures as described in equations 28 to 31.

3.2. Real Data Application

The ARMA-GARCH-Normal models of equations 22 and 23, the Suaiso-Mapa methodology as shown in equations 26 and 27, and the proposed ARMA-GARCH-Folded EVT equations 32 and 33 were fitted and evaluated on stock indices and exchange rates data. The VaR and ES were solved with 1 α = 0.99. Three daily stock indices data were used: (1) the Philippine Stock Exchange Index (PSEI), (2) the Dow Jones Industrial Average (DJIA), and (3) the S&P 500 Index (SNP). The daily stock indices data were gathered from Yahoo Finance (https://finance.yahoo.com/). Two daily currency exchange rates data were used: (1) the Philippine Peso-US Dollar Exchange Rate (PHPUSD), and (2) the Philippine Peso-Euro Exchange Rate (PHPEUR). All time series data span from 30 October 2006 to 31 July 2018. The daily currency rates data were gathered from the BSP [6]. The log-returns as defined in equation 3 were computed for each time series in all covered time points. Each of the stock index data series were divided to two sets of periods: (1) the training period were all dates before 1 August 2017, and (2) the testing or evaluation period from 1 August 2017 to 31 July 2018. The currency rates data series were divided to two sets of periods: (1) the training period were all dates before 24 March 2017, and (2) the testing or evaluation period from 24 March 2017 to 31 July 2018. For both types of data, this ensured that there are 251 time periods for the testing interval. There are many skips in exchange rate data because of international date matching for cross rates data. Missing rates or index values in these series are due to holidays, and thus were ignored. The ARMA models were selected using the auto.arima function in R [20]. All models have GARCH(1,1) variance specifications. For the POT models, the Hill plots were used for threshold selection for each series. All stated 99 percent VaR and 99 percent ES models were tested using equations 28 to 30. The arithmetic averages, quartiles, and maximum VaR-based loss values as stated in equation 31 were compared among the different VaR and ES models.

4. Results and Discussion

4.1. Summary Statistics of Financial Time Series

The level and return plots of the five time series data are shown below. It is noted that the volatility in the stock indices were highest during the 2008-2009 Global Financial Crisis which can be found in the first few periods in the return plots. The currency rates series are relatively less volatile with the ranges of the returns not necessarily going beyond double digits in their daily fluctuations.

Figure 1: Level and Return Plots for PSEI, 30 October 2006 - 31 July 2017

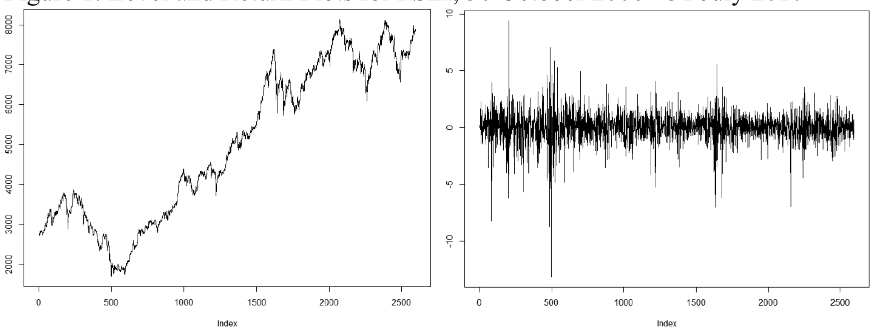


Figure 2: Level and Return Plots for DJIA, 30 October 2006 - 31 July 2017

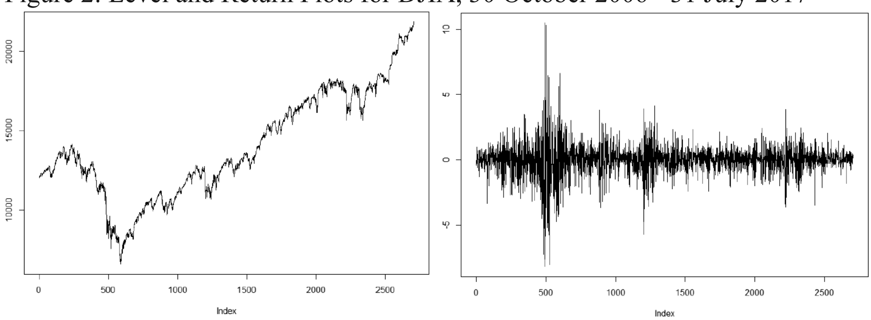


Figure 3: Level and Return Plots for SNP, 30 October 2006 - 31 July 2017

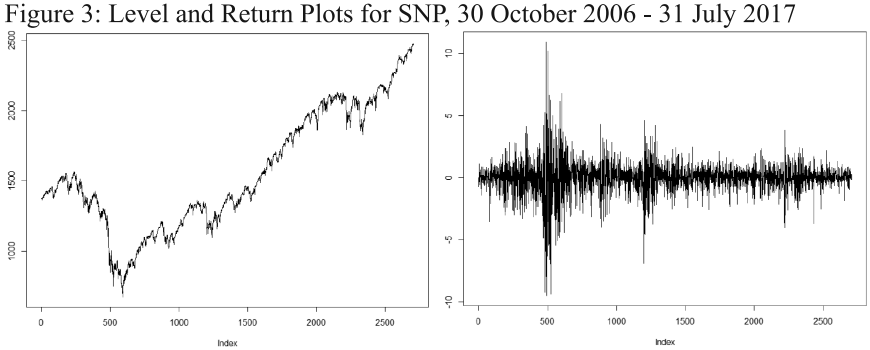


Figure 4: Level and Return Plots for PHPUSD, 30 October 2006 - 23 March 2017

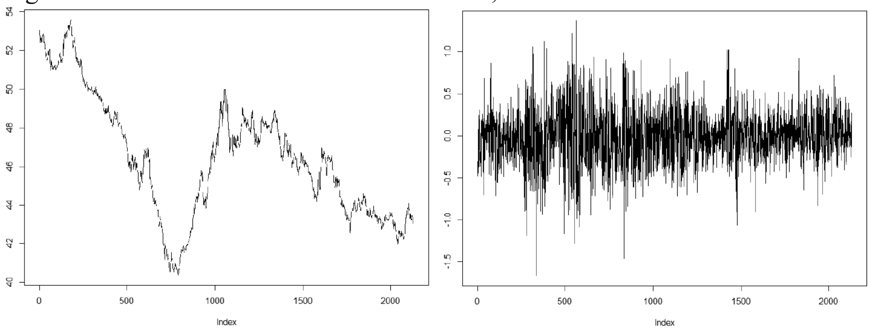


Figure 5: Level and Return Plots for PHPEUR, 30 October 2006 - 23 March 2017

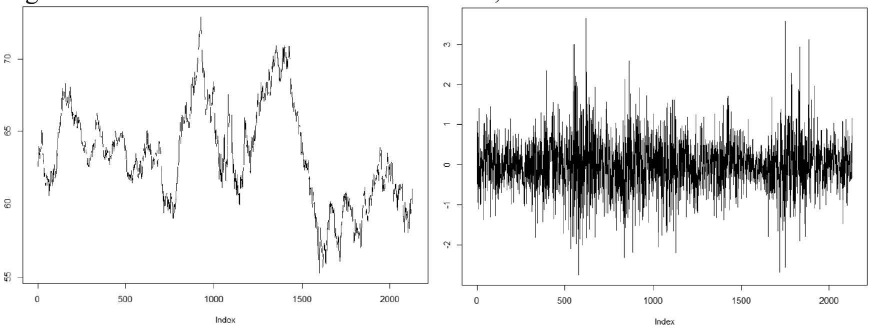


Table 1 below shows the basic statistics of the returns of time series under consideration. For the stocks data, they tend to be negatively skewed with high kurtosis values. For the currency returns, skewness is positive and kurtosis, though higher than 3, indicate that their tails are not as heavy as the stock indices datasets.

Table 1: Summary Statistics of Returns Data on the Training Periods

	PSEI	DJIA	SNP	PHPUSD	PHPEUR
Minimum	-13.08869	-8.20051	-9.46951	-1.665039	-2.73432
First Quartile	-0.57239	-0.40507	-0.41481	-0.21326	-0.38962
Median	0.07341	0.04840	0.05783	0.00000	0.00702
Third Quartile	0.72602	0.52615	0.56383	0.18867	0.39492
Maximum	9.36528	10.50835	10.95720	1.36768	3.64271
Mean	0.03940	0.02207	0.02166	-0.00692	0.00534
Standard Deviation	1.29702	1.17368	1.27831	0.33315	0.67650
Skewness	-0.81395	-0.10327	-0.34244	0.01426	0.23038
Kurtosis	11.92516	13.54823	13.65691	3.97883	4.94880
Jarque-Bera test	8909.89664	12549.95634	12857.87336	85.14394	356.06591
JB p-value	0.00000	0.00000	0.00000	0.00000	0.00000
Sample	2598	2706	2706	2131	2131

4.2. Model Fitting

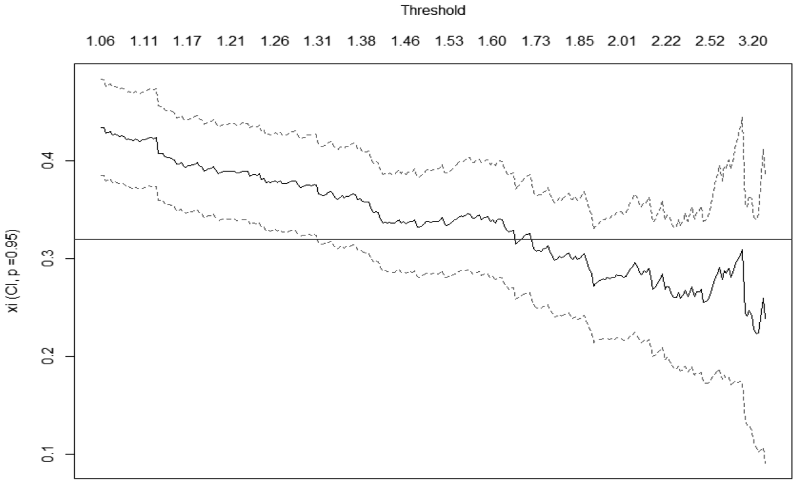
The results of fitting an ARMA-GARCH model for each of the five time series data are shown below in table 2. The ARMA order is selected by the auto. arima function based on AIC, so some results in the lags may not be significantly different from zero. The GARCH model parameters are all significant as indicated by the Standard Errors (SEs), which indicate nonconstant heteroskedasticity which is typical of financial time series.

Table 2: ARMA-GARCH Parameter Estimates of Returns on Training Periods

	PSEI	DJIA	SNP	PHPUSD	PHPEUR
ARMA order p	2	2	0	1	2
MA order q	2	4	2	0	2
ϕ_1	0.98241	0.40976	–	0.09643	0.81165
robust se	0.27858	0.13080	–	0.02117	0.17190
ϕ_2	-0.36791	-0.85413	–	–	-0.34700
robust se	0.21358	0.09159	–	–	0.15866
θ_1	-0.88085	-0.46645	-0.06712	–	-0.90375
robust se	0.29011	0.13160	0.01897	–	0.16302
θ_2	0.26673	0.88664	-0.00663	–	0.45178
robust se	0.21388	0.09038	0.02236	–	0.14723
θ_3	–	-0.02804	–	–	–
robust se	–	0.02445	–	–	–
θ_4	–	-0.01302	–	–	–
robust se	–	0.02767	–	–	–
α_0	0.05726	0.02324	0.02474	0.00240	0.00238
robust se	0.02470	0.00542	0.00633	0.00092	0.00123
α_1	0.14171	0.12295	0.11661	0.10326	0.04166
robust se	0.02496	0.01925	0.01917	0.01903	0.00488
β_1	0.82914	0.85607	0.86370	0.87573	0.95338
robust se	0.03221	0.01906	0.01893	0.02447	0.00372

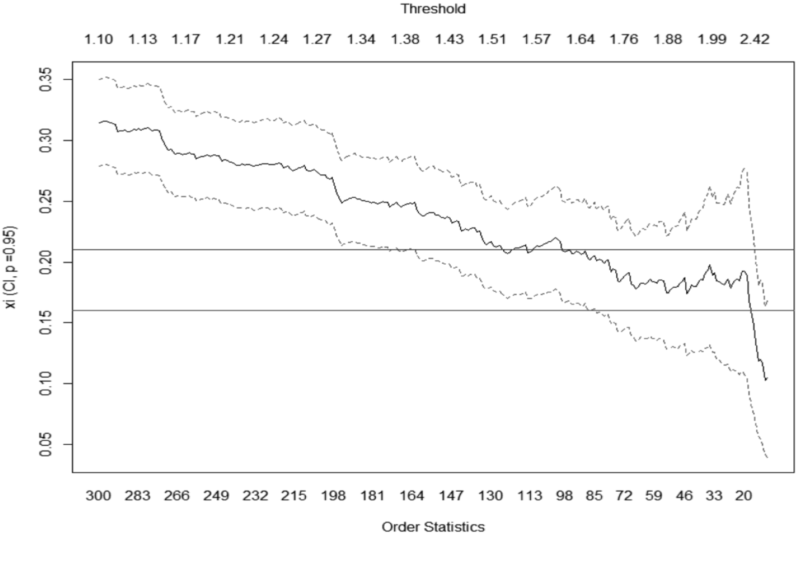
Figure 6 below shows the Hill plot for the standardized residuals of the ARMA- GARCH model for PSEI. The blue line is marked on $\xi = 0.32$. Note that since ξ is a constant, it should stay constant, or in the case of the graph, lie on the horizontal line starting from the right end of the graph such as shown in the blue line. The lower horizontal axis indicates the kth largest value in the sample $X_{(n-k)}$ that was used as the threshold to solve for $\hat{xi}_{Hill,k}$. The upper horizontal axis simply shows the numerical threshold value each $X_{(n-k)}$. Based on the figure below, an appropriate threshold would be $k^* \leq 190$. The threshold chosen as appropriate was $k^* = 150$ thus $\hat{\delta} = 1.51093\%$. For folding, the initial threshold was selected at $k_F = 190$ which is $\hat{\delta} = 1.36129\%$. It is noted that these chosen values are slightly subjective within the choice $k^* \leq 190$ but in the literature of threshold choice for POT models, there is no definitive means to choose.

Figure 6: Hill Plots for PSEI ARMA-GARCH Standardized Residuals



The other Hill plots for standardized residuals follow the same explanation for threshold choice, except for standardized residuals of the ARMA-GARCH model for PHPUSD as shown in figure 7, in which some level of discretion has been exercised. The blue line is drawn at $\xi = 0.21$ but another purple line was drawn at $\xi = 0.16$. To base the choice on $\xi = 0.16$ is suboptimal as too few observations will be the basis for the parameter estimates of the folding transformation, which will depend on the initial threshold choice. Researcher discretion has been exercised to ignore the ξ values from $k \leq 30$. This meant that $k^* \leq 150$ would be appropriate. The threshold at $k^* = 120$ was chosen, indicating that $\hat{\delta} = 1.54566\%$

Figure 7: Hill Plots for PHPUSD ARMA-GARCH Standardized Residuals



For the sake of brevity, the other Hill plots will be displayed below without further explanation as they will follow the same form as it is done in the PSEI data. The choice of appropriate threshold and pre-folding threshold are shown in table 3.

Figure 8: Hill Plots for DJIA ARMA-GARCH Standardized Residuals

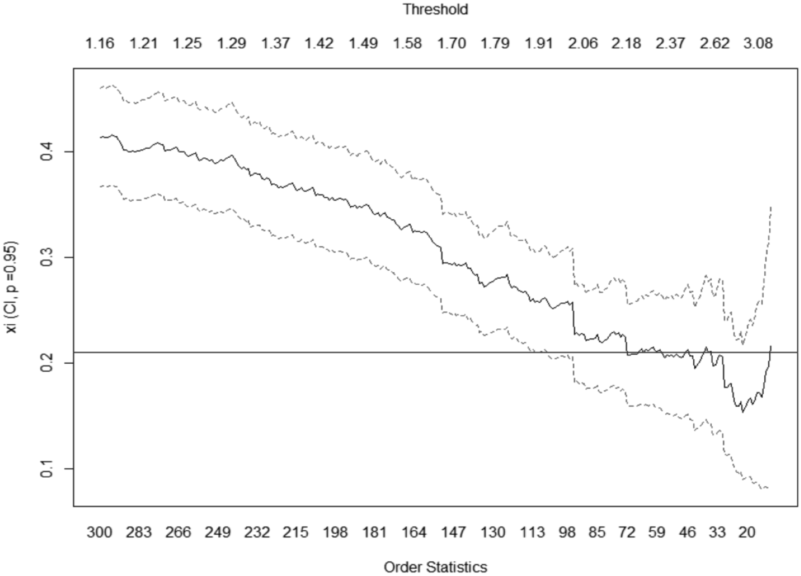


Figure 9: Hill Plots for SNP ARMA-GARCH Standardized Residuals

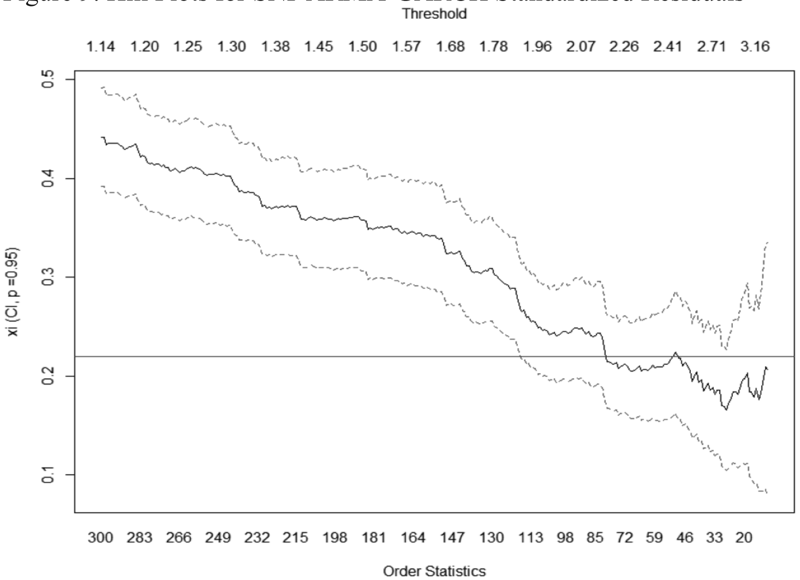


Figure 10: Hill Plots for PHPEUR ARMA-GARCH Standardized Residuals

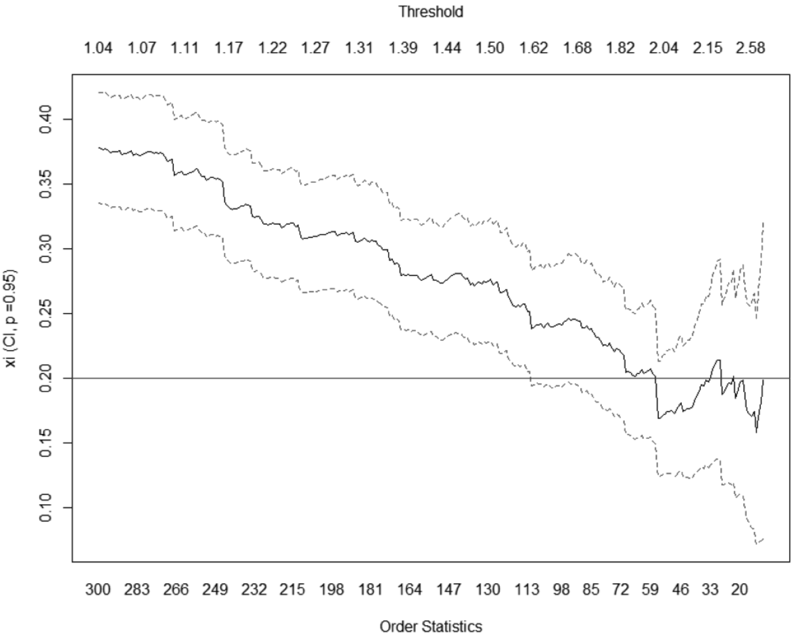


Table 3: Threshold Choices for ARMA-GARCH Standardized Residuals

	PSEI	DJIA	SNP	PHPUSD	PHPEUR
k_F	190	90	110	150	100
$\tilde{\delta}$	1.36129%	2.07031%	1.96025%	1.43141%	1.65588%
k^*	150	70	85	120	50
$\hat{\delta}$	1.51093%	2.23209%	2.11824%	1.54566%	2.05235%

Table 4 below shows the MLE results for the EVT models for pre-fold, post-fold, and unfolded procedures. In the unfolded results, all $\hat{\xi}$ estimates indicate that ξ not significantly different from zero as it can be checked by the SEs. For the post-fold results, only PSEI and DJIA have $\hat{\xi}$ estimates indicating significant nonzero ξ values. This means that PSEI and DJIA have heavy tails while the three other series have exponential tails, which are similar to the tails from the normal distribution.

Table 4: EVT Results for Pre-Fold, Post-Fold, and Unfolded Procedures

	PSEI	DJIA	SNP	PHPUSD	PHPEUR
Pre-fold					
$\tilde{\xi}_{ML,\tilde{\delta}}$	0.13412	0.08723	0.04394	-0.00233	0.04190
se	0.08228	0.11460	0.09954	0.08146	0.10419
$\tilde{\sigma}_{ML,\tilde{\delta}}$	0.62002	0.54441	0.61244	0.42514	0.48435
se	0.06774	0.08462	0.08438	0.04903	0.06992
Post-fold					
$\hat{\xi}_{FML,\hat{\delta}}$	0.12979	0.08291	0.03853	-0.00552	0.03984
se	0.02217	0.02082	0.01997	0.02155	0.02253
$\hat{\sigma}_{FML,\hat{\delta}}$	0.64141	0.55932	0.62176	0.42410	0.49930
se	0.01892	0.01583	0.01723	0.01296	0.01560
Unfolded					
$\hat{\xi}_{ML,\hat{\delta}}$	0.15226	0.13899	0.04616	0.06346	0.30395
se	0.09408	0.13614	0.11380	0.09708	0.18441
$\hat{\sigma}_{ML,\hat{\delta}}$	0.62241	0.50454	0.61488	0.37583	0.32001
se	0.07715	0.09102	0.09661	0.05003	0.07308

4.3. Risk Measurement Performance

The risk estimation performance of the different 99% VaR and 99% ES models with respect to exceedances and the Christoffersen likelihood tests are shown in table 5 below. Note that for most of the returns series, the maximum number of exceedances that ARMA-GARCH (AG) folded POT (FPOT) has received was 1 in the PHPUSD currency rate series. Because of this, the likelihood tests cannot be performed for the AGFPOT except in the PHPUSD series, in which, AGFPOT is assured to meet the required coverage, have serially independent exceedances, and conditional exceedance occur in appropriate coverage proportions. The AGPOT model, which is the Suaiso-Mapa methodology, may have some independence problems as the VaR in DJIA and SNP and the ES in DJIA indicate that the exceedances are serially correlated. This means that for these series, it is possible that past exceedances may indicate possible following exceedances. The AG-Normal methodology consistently has higher or same exceedances than the AGPOT, while AGPOT have higher or same exceedances than the AGFPOT.

Table 5: Exceedances and Likelihood Tests for 99% VaR and 99% ES Models

	VaR					ES				
	PSEI	DJIA	SNP	PHPUSD	PHPEUR	PSEI	DJIA	SNP	PHPUSD	PHPEUR
Exceedances										
AG Normal	1	5	7	1	2	0	3	4	1	1
AGPOT	0	3	4	1	2	0	2	1	1	0
AGFPOT	0	0	0	1	0	0	0	0	1	0
LR_{uc}										
AGNormal	1.189	1.937	5.460	1.189	0.113	–	0.091	0.757	1.189	1.189
p-value	0.276	0.164	0.019	0.276	0.737	–	0.763	0.384	0.276	0.276
AGPOT	–	0.091	0.757	1.189	0.113	–	0.113	1.189	1.189	–
p-value	–	0.763	0.384	0.276	0.737	–	0.737	0.276	0.276	–
AGFPOT	–	–	–	1.189		–	–	–	1.189	–
p-value	–	–	–	0.276	1.000	–	–	–	0.276	–
LR_{ind}										
AG Normal	0.008	3.161	1.852	0.008	0.032	–	5.433	4.115	0.008	0.008
p-value	0.929	0.075	0.174	0.929	0.857	–	0.020	0.043	0.929	0.929
AGPOT	–	5.433	4.115	0.008	0.032	–	7.502	0.008	0.008	–
p-value	–	0.020	0.043	0.929	0.857	–	0.006	0.929	0.929	–
AGFPOT	–	–	–	0.008	0.000	–	–	–	0.008	–
p-value	–	–	–	0.929	1.000	–	–	–	0.929	–
LR_{cc}										
AG Normal	1.197	5.098	7.312	1.197	0.145	–	5.524	4.872	1.197	1.197
p-value	0.550	0.078	0.026	0.550	0.930	–	0.063	0.088	0.550	0.550
AGPOT	–	5.524	4.872	1.197	0.145	–	7.614	1.197	1.197	–
p-value	–	0.063	0.088	0.550	0.930	–	0.022	0.550	0.550	–
AGFPOT	–	–	–	1.197	–	–	–	–	1.197	–
p-value	–	–	–	0.550	–	–	–	–	0.550	–

Table 6 shows the results for the evaluation of VaR-based loss function. Generally, AGFPOT would tend to have higher minimum, quartile, and mean losses than the other models over all other models in all return series. However, except for the PSEI in the VaR, the maximum loss is lowest for the AGFPOT over the two other models. This indicates that generally, AGFPOT measures risk generally higher than the other models, but when extreme losses are observed, the AGFPOT is able to account compared to the other models. These other models are less often to cover extreme losses than the AGFPOT. This indicates that the AGFPOT is a more conservative measure of risk, asking institutions which would use this model to have more risk capital than the other two models with the added benefit of ensured and strengthened cushion over extreme losses than the two models can provide.

Table 6: VaR-Based Loss Function $\tilde{Q}$ for 99% VaR and 99% ES Models

	VaR					ES				
	PSEI	DJIA	SNP	PHPUSD	PHPEUR	PSEI	DJIA	SNP	PHPUSD	PHPEUR
AG Normal										
Minimum	0.179	0.026	0.286	0.035	0.320	0.247	0.184	0.145	0.120	0.068
1st Quartile	1.645	1.293	1.279	0.430	0.926	1.923	1.486	1.465	0.518	1.104
Median	2.286	1.714	1.669	0.601	1.231	2.616	1.924	1.888	0.677	1.401
3rd Quartile	2.888	2.292	2.348	0.762	1.619	3.261	2.563	2.587	0.870	1.791
Maximum	5.638	189.735	170.185	34.824	27.187	6.089	152.079	136.722	28.756	9.488
Mean	2.315	3.484	3.173	0.746	1.409	2.636	3.339	2.957	0.806	1.469
AGPOT										
Minimum	0.329	0.236	0.246	0.011	0.051	1.149	0.058	0.201	0.124	0.060
1st Quartile	1.990	1.529	1.566	0.408	0.947	2.847	1.937	1.948	0.522	1.259
Median	2.710	1.968	2.007	0.580	1.250	3.634	2.424	2.466	0.680	1.549
3rd Quartile	3.370	2.623	2.725	0.734	1.639	4.390	3.223	3.219	0.875	1.939
Maximum	6.218	143.242	119.243	36.510	25.226	7.494	68.928	51.688	28.492	2.810
Mean	2.732	3.315	2.881	0.729	1.412	3.688	3.138	3.010	0.809	1.585
AGFPOT										
Minimum	2.666	1.305	1.010	0.318	0.912	3.503	1.961	1.372	0.409	1.242
1st Quartile	4.462	3.073	2.942	0.693	2.084	5.646	3.578	3.386	0.780	2.401
Median	5.329	3.653	3.664	0.870	2.393	6.609	4.243	4.124	0.963	2.728
3rd Quartile	6.437	4.959	4.627	1.100	2.794	7.964	5.842	5.328	1.217	3.119
Maximum	10.304	13.790	12.196	14.267	3.811	12.453	15.792	13.733	6.903	4.209
Mean	5.514	4.385	4.149	0.950	2.435	6.849	5.105	4.752	1.023	2.763

5. Conclusion

In the activities of financial institutions with interactions with financial markets, such as buying and selling assets, they incur risks in abrupt changes in the value of held assets. Since the quality of service and lifetime financial institutions depend on the value of held assets, financial regulators have opted that these institutions manage the risk they accumulate by preparing an appropriate amount of risk capital. The capital is computed based on risk measures, such as VaR and ES via the internal models approach.

However, to model for VaR and ES, an understanding of the stylized facts of time series is important. It is generally known that financial time series data have non-normal properties such as skewness and high kurtosis indicating thick tails and that variance tends to vary through time. Thus, these features need to be accounted in modeling and estimating risk.

As market risks involve extremely high losses, it is best to key in to the tails of a distribution in determining risk, thus the POT models have been devised, which are based on the GPD distribution of EVT. The problem of POT models is the high sample requirement due to chosen threshold that defines the tails and the deletion of data when found not on the tails. Data deletion results into loss of information that are useful in the estimation of GPD parameters. Thus, folding in POT models are used to improve estimation of the parameters to facilitate better estimation of extreme quantiles necessary for risk measurement.

The paper outlines a method of estimating VaR and ES using AGFPOT models. The risk measures based on the folded POT are compared to the ARMA- GARCH Normal model and the Suaiso-Mapa methodology in terms of the performance on exceedances and the VaR-based loss function $\tilde{Q}$. On the generated statistics, the AGFPOT model is a conservative risk model in which exceedances are observed less often than the two other models and though the AGFPOT gives higher VaR and ES values, it provides adequate risk capital for extreme losses compared to large losses possible in exceedances with the two other models. The paper shows the viability of the AGPOT model for risk estimation and measurement for financial institutions with reduced exceedances and conservative risk capital computation.

References

[1] Acerbi, C, and Tasche, D, Expected Shortfall: a natural coherent alternative to Value at Risk, *Economic Notes*, vol. 31, issue 2, pp. 379-388, 2002.

[2] Akaike, H, A new look at the statistical model identification, *IEEE Transactions on Automatic Control*, vol. 19, issue 6, pp. 716-723, 1974.

[3] Artzner, P; Delbaen, F, Eber, JM, and Heath, D, Thinking Coherently, *Risk*, vol. 10, pp. 68-71, 1997.

[4] Balkema, A., and De Haan, L, Residual lifetime at great age, *Annals of Probability*, vol. 2, issue 5, pp. 792-804, 1974.

[5] Bangko Sentral ng Pilipinas, *Circular No. 538: Revised Risk-Based Capital Adequacy Framework*, 2006.

[6] Bangko Sentral ng Pilipinas, *Peso, US Dollar and Euro Crosss Rates - Daily, http://www.bsp.gov.ph/statistics/excel/dailycrossrates.xls*, accessi- ble as of 20 August 2018, 2006.

[7] Bollerslev, T, Generalized Autregressive Conditional Heteroskedasticity, *Journal of Econometrics*, vol. 31, pp. 307-327, 1986.

[8] Box, G, and Jenkins, G, *Time Series Analysis: Forecasting and Control*, San Francisco: Holden-Day, 1970.

[9] Casella, G, and Berger, RL, *Statistical Inference*, Pacific Grove, CA, USA: Duxbury Thomson Learning, 2002.

[10] Coles, S, *An Introduction to Statistical Modeling of Extreme Values*, London, UK: Springer-Verlag, 2001.

[11] Corcoran, JN, and Schneider, U, Shift and scale coupling methods for perfect simulation, *Probability in the Engineering and Informational Sciences*, vol. 17, pp. 227-303, 2003.

[12] Christoffersen, P, Evaluating interval forecast, *International Economic Review*, vol. 3, issue 4, pp. 841-862, 1998.

[13] De Haan, L, and Ferreira, A, *Extreme Value Theory: An Introduction*, New York, NY: Springer Science+Business Media, LLC, 2006.

[14] Diebold, FX, *Elements of Forecasting: Fourth edition*, Mason, OH: Thomson South-Western, 2007.

[15] Engle, RF, Autoregressive Conditional Heteroskedastcity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, vol. 50, pp. 987-1008, 1982.

[16] Fama, E, The behavior of stock market prices, *The Journal of Business*, vol. 38, pp. 34-105, 1965.

[17] Gonzalez-Rivera, G; Lee, TH; and Mishra, S, Forecasting volatility: A reality check based on option pricing, utility function, Value-At-Risk, and predictive likelihood *International Journal of Forecasting*, vol. 20, pp. 629-645, 2004.

[18] Guillou, A; Naveau, P, and You, A, A folding method for extreme quantiles estimation *REVSTAT-Statistical Journal*, vol. 8, no. 1, pp. 21-35, 2010.

[19] Hill, BM, A simple general approach to inference about the tail of a distribution, *Annals of Statistics*, vol. 3, pp. 1163-1174, 1975.

[20] Hyndman, RJ, and Khandakar, Y, Automatic Time Series Forecasting: the forecast Package in R, *Journal of Statistical Software*, vol. 27, issue 3, pp. 1-22, 2008.

[21] Jorion, P, *Value at Risk: The New Benchmark for Managing Financial Risk, 3rd Ed.*, USA: McGraw-Hill, 2007.

[22] Ljung, GM, and Box, GEP, On a Measure of a Lack of Fit in Time Series Models, *Biometrika*, vol. 65, issue 2, pp. 297-303, 1978.

[23] Mandelbrot, B, The variation of certain speculative prices, *The Journal of Business*, vol. 35, pp. 394-419, 1963.

[24] McNeil, AJ; Frey, R and Embrechts P, *Quantitative Risk Management: Concepts, Techniques, Tools*, New Jersey: Princeton University Press, 2005.

[25] Pickands, J, Statistical inference using extreme order statistics, *Annals of Statistics*, vol. 3, pp. 119-131, 1975.

[26] Suaiso, JOQ, and Mapa, DS, Measuring market risk using extreme value theory, *Philippine Review of Economics*, vol. 46, issue 2, pp. 91-121, 2009.

[27] Tsay, RS, *Analysis of Financial Time Series*, New York, NY: John Wiley and Sons, Inc., 2002.

[28] White, H, Maximum likelihood estimation of misspecified models, *Econometrica*, vol. 50, pp. 1-25, 1982.