A General Class of Chain Ratio-Product Type Exponential Estimators in Double Sampling using Two Auxiliary Variates

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In this paper, a general class of chain ratio-product type exponential estimators has been proposed for estimating a finite population mean in presence of two auxiliary variates under double sampling scheme. The expressions for bias and mean square error (MSE) of the proposed class are derived up to the first degree of approximation. Also, the expression of asymptotic optimum estimator (AOE) in the proposed class is obtained. Some estimators are shown to be particular members of the proposed class. The proposed class has been compared for its precision with the usual unbiased estimator and several other estimators of the literature. In addition, an empirical study is also carried out in support of theoretical findings.

Keywords: auxiliary variates, study variate,double sampling, bias, mean square error

1. Introduction

The estimation of population parameters (such as the mean and/or variance) of a study variate is a common issue in sample surveys. Utilizing information on auxiliary variates, a great variety of techniques have been developed in sampling literature for improving the efficiencies of the estimators of population parameters.

Ratio, product and regression methods of estimation are good examples in this context and are widely employed in many situations of practical importance.

A ratio estimator for estimating the population mean $\bar{Y}$ of a study variate Y, by utilizing the population mean $\bar{X}$ of an auxiliary variate X, which is positively correlated with Y was developed by Cochran (1940). Robson (1957) defined a product estimator for $\bar{Y}$ that was revisited by Murthy (1964). The product estimator $\bar{Y}$ is preferred when the correlation coefficient between the variates Y and X (i.e., ρ_{yx}) is negative. However, in many situations, the population mean $\bar{X}$ of the auxiliary variate X is not known in advance. In such a situation, double sampling technique is adopted. Neyman (1938) was the first to give the concept of double sampling in connection with collecting information on the strata size in a stratified sampling. The problem of double sampling has been studied by various authors. For further information on applications of double sampling methods the reader is referred to Hidiroglou and Sarndal (1998), Fuller (2000), and Hidiroglou (2001). We can use one, two or more auxiliary variates in estimating population mean. Keeping this fact in view, Chand (1975) introduced chain ratio type estimators for the population mean $\bar{Y}$. This led several authors including Kiregyera (1980), Singh and Upadhyaya (1995), Prasad et al. (1996), Singh and Choudhury (2012), Vishwakarma and Gangele (2014), and Vishwakarma and Kumar (2014) to modify the chain type estimators and discuss their properties.

Let $U=\{U_1, U_2,\ldots, U_N\}$ be a finite population of size N. Also, let Y and X be the study and auxiliary variates taking the values y_i and x_i respectively, on the unit U_i ($i = 1, 2,\ldots, N$). For estimating the population mean $\bar{Y}$ in double sampling, a first phase sample of size n' is selected from the population U and observations are made only on the variate X. Subsequently, a second phase sample of size n is selected from the first phase sample on which both the variates Y and X are observed.

Sometimes, information on another auxiliary variate Z, which is closely related to X but compared to X remotely related to Y, is available on all units of the population. Let $\bar{Z}$ be the known population mean of Z and $C\rho_{xz}C_x/C_z > 1/2$, where C_x, C_z and ρ_{xz} denote, respectively, the coefficient of variation of X, the coefficient of variation of Z, and the correlation coefficient between X and Z. Then in such a situation the ratio estimator $\bar{x}'\bar{Z}/\bar{z}'$ will estimate $\bar{X}$ more precisely than $\bar{x}'$ to terms of order $0(n^{-1})$. Using this approach Chand (1975) suggested a chain ratio estimator for $\bar{Y}$ in double sampling as

$$\bar{y}_R^{(C)} = \bar{y}\frac{\bar{x}'}{\bar{x}}\frac{\bar{Z}}{\bar{z}'} \tag{1.1}$$

2. Proposed Class of Estimators

We define a class of chain ratio-product type exponential estimators for the population mean $\bar{Y}$ in double sampling as

$$T = \bar{y}\left[k\exp\left\{\frac{\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)-\bar{x}}{\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)+\bar{x}}\right\}+(1-k)\exp\left\{\frac{\bar{x}-\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)}{\bar{x}+\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)}\right\}\right] \quad (2.1)$$

where, α and β denote the scalars, k is a real constant to be determined so as to minimize the *mean square error (MSE)* of the proposed class T.

To obtain the *bias* and *MSE* of the proposed class of estimators T, we consider

$$\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1), \bar{x}' = \bar{X}(1+e_1'), \bar{z}' = \bar{Z}(1+e_2')$$

such that

$$E(e_0) = E(e_1) = E(e_1') = E(e_2') = 0$$

Also, we have

$$\left.\begin{aligned}
&E(e_0^2) = f_1C_y^2, E(e_1^2) = f_1C_x^2, E(e_1'^2) = f_2C_x^2, E(e_2'^2) = f_2C_z^2\\
&E(e_0e_1) = f_1\rho_{yx}C_yC_x, E(e_0e_1') = f_2\rho_{yx}C_yC_x, E(e_0e_2') = f_2\rho_{yz}C_yC_z\\
&E(e_1e_1') = f_2C_x^2, E(e_1e_2') = f_2\rho_{xz}C_xC_z, E(e_1'e_2') = f_2\rho_{xz}C_xC_z
\end{aligned}\right\} \quad (2.2)$$

where,

$$f_1 = \left(\frac{1}{n}-\frac{1}{N}\right), f_2 = \left(\frac{1}{n'}-\frac{1}{N}\right), f_3 = f_1 - f_2 = \left(\frac{1}{n}-\frac{1}{n}\right)'$$

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2}, C_z^2 = \frac{S_z^2}{\bar{Z}^2}, \rho_{yx} = \frac{S_{yx}}{S_yS_x}, \rho_{yz} = \frac{S_{yz}}{S_yS_z}, \rho_{yx} = \frac{S_{xz}}{S_xS_z},$$

$$S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(y_i-\bar{Y})^2,\ S_x^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(x_i-\bar{X})^2, S_z^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(z_i-\bar{Z})^2,$$

$$S_{yx}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right), S_{yz}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(z_i-\bar{Z}\right),$$

$$S_{xz}=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)\left(z_i-\bar{Z}\right).$$

Now, expressing T in terms of $e's$, we have

$$T=\bar{Y}\left(1+e_0\right)\left[k\exp\left(U_1\right)+\left(1-k\right)\exp\left(U_2\right)\right] \tag{2.3}$$

where

$$U_1=\frac{1}{2}\left(e_1'-\lambda_1e_2'-e_1-\lambda_1e_1e_2'\right)\left\{1+\frac{1}{2}\left(e_1'+\lambda_1e_2'+e_1+\lambda_1e_1e_2'\right)\right\}^{-1}$$

$$U_2=\frac{1}{2}\left(\lambda_1e_2'+e_1+\lambda_1e_1e_2'-e_1'\right)\left\{1+\frac{1}{2}\left(e_1'+\lambda_1e_2'+e_1+\lambda_1e_1e_2'\right)\right\}^{-1};\lambda_1=\frac{\alpha\bar{Z}}{\alpha\bar{Z}+\beta}$$

Expanding the right hand side of (2.3), multiplying out and retaining the terms of $e's$ up to the second degree, we obtain

$$\begin{aligned}T=\bar{Y}\Bigg[&1+k(e_1'-\lambda_1e_2'-e_1+e_0e_1'-\lambda_1e_0e_2'-e_0e_1)-\frac{k}{2}(e_1'^2-\lambda_1^2e_2'^2-e_1^2)\\&+e_0-\frac{1}{2}\left(e_1'-\lambda_1e_2'-e_1+e_0e_1'-\lambda_1e_0e_2'-e_0e_1\right)\\&+\frac{1}{4}\left(e_1'^2-\lambda_1^2e_2'^2-e_1^2-\lambda_1e_1'e_2'+\lambda_1e_1e_2'-e_1e_1'\right)+\frac{1}{8}(e_1'^2+\lambda_1^2e_2'^2+e_1^2\Bigg]\end{aligned}$$

or

$$\begin{aligned}T-\bar{Y}=\bar{Y}\Bigg[&k(e_1'-\lambda_1e_2'-e_1+e_0e_1'-\lambda_1e_0e_2'-e_0e_1)-\frac{k}{2}\left(e_1'^2-\lambda_1^2e_2'^2-e_1^2\right)\\&+e_0-\frac{1}{2}\left(e_1'-\lambda_1e_2'-e_1+e_0e_1'-\lambda_1e_0e_2'-e_0e_1\right)\\&+\frac{1}{4}\left(e_1'^2-\lambda_1^2e_2'^2-e_1^2-\lambda_1e_1'e_2'+\lambda_1e_1e_2'-e_1e_1'\right)+\frac{1}{8}\left(e_1'^2+\lambda_1^2e_2'^2+e_1^2\right)\Bigg]\end{aligned} \tag{2.4}$$

Taking the expectation in (2.4) and using results in (2.2), we get the *bias* of the class T to the first degree of approximation as

$$B(T)=\bar{Y}\left[\frac{(4k-1)}{8}\left\{f_3C_x^2+\lambda_1^2 f_2C_z^2\right\}-\frac{(2k-1)}{2}\left\{f_3\rho_{yx}C_yC_x+\lambda_1 f_2\rho_{yz}C_yC_z\right\}\right] \quad (2.5)$$

Again from (2.4), by neglecting the terms of e's having degree greater than one, we have

$$T-\bar{Y}=\bar{Y}\left[k\left(e_1'-\lambda_1 e_2'-e_1\right)+e_0-\frac{1}{2}\left(e_1'-\lambda_1 e_2'-e_1\right)\right] \quad (2.6)$$

Squaring both sides of (2.6), taking the expectation and using results in (2.2), we obtain the MSE of the class T to the first degree of approximation as

$$MSE(T)=\bar{Y}^2\left[f_1C_y^2+\frac{(2k-1)^2}{4}\left\{f_3C_x^2+\lambda_1^2 f_2C_z^2\right\}-(2k-1)\left\{f_3\rho_{yx}C_yC_x+\lambda_1 f_2\rho_{yz}C_yC_z\right\}\right] \quad (2.7)$$

2.1 Asymptotic optimum estimator

Minimization of *MSE (T)* in (2.7) with respect to λ_1 yields the optimum value of λ_1 as

$$\left(\lambda_1\right)_{opt}=\frac{2\rho_{yz}C_y}{(2k-1)C_z} \quad (2.8)$$

Substituting (2.8) in (2.7) and minimizing again the expression of *MSE(T)*, so obtained, with respect to *k*, we get the optimum value of *k* as

$$(k)_{opt}=\frac{1}{2}\left(\frac{2\rho_{yx}C_y}{C_x}+1\right) \quad (2.9)$$

Hence the minimum attainable *MSE* of the class *T* is given by

$$MSE(T)_{\min}=\bar{Y}^2C_y^2\left(f_1-f_3\rho_{yx}^2-f_2\rho_{yz}^2\right) \quad (2.10)$$

Substituting (2.9) in (2.1), we get the *asymptotic optimum estimator (AOE)* as

$$T_{opt}=\bar{y}\left[\left(\frac{\rho_{yx}C_y}{C_x}+\frac{1}{2}\right)exp\left\{\frac{\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}\left(\alpha\bar{Z}+\beta\right)-\bar{x}}{\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}\left(\alpha\bar{Z}+\beta\right)+\bar{x}}\right\}\right.$$

$$+\left(\frac{1}{2}-\frac{\rho_{yx}C_y}{C_x}\right)exp\left\{\frac{\bar{x}-\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)}{\bar{x}+\frac{\bar{x}'}{(\alpha\bar{z}'+\beta)}(\alpha\bar{Z}+\beta)}\right\}\Bigg] \tag{2.11}$$

Remark 2.1. For $\frac{\rho_{yx}C_y}{C_x}=\frac{1}{2}$, the *asymptotic optimum estimator (AOE)* in (2.11) reduces to the class of chain ratio-type exponential estimators, as given in Vishwakarma and Gangele (2014).

3. Members of the Proposed Class of Estimators

In Table 1, some of the members of the class *T* have been shown. These members are obtained by assigning suitable values to the constants *k*, *α* and *β* in (2.1). The expressions for the bias and MSE of these members can be obtained by mere substituting the values of *k*, *α* and *β* in (2.5) and (2.7), respectively. It has also been verified, with the help of an empirical study, that these members are not optimum in the proposed class.

Table 1. Members of the Class of Estimators T

Authors	Estimators	k	α	β
Singh and Vishwakarma (2007)	$\hat{\bar{Y}}_{ReMd}=\bar{y}\ exp\left(\frac{\bar{x}'-\bar{x}}{\bar{x}'+\bar{x}}\right)$	1	0	1
	$\hat{\bar{Y}}_{PeMd}=\bar{y}\ exp\left(\frac{\bar{x}-\bar{x}'}{\bar{x}+\bar{x}'}\right)$	0	0	1
Singh and Choudhury (2012)	$\hat{\bar{Y}}^{dc}_{Re}=\bar{y}\ exp\left\{\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z}-\bar{x}\right)\Big/\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z}+\bar{x}\right)\right\}$	1	1	0
	$\hat{\bar{Y}}^{dc}_{Pe}=\bar{y}\ exp\left\{\left(\bar{x}-\frac{\bar{x}'}{\bar{z}'}\bar{Z}\right)\Big/\left(\bar{x}+\frac{\bar{x}'}{\bar{z}'}\bar{Z}\right)\right\}$	0	1	0
Vishwakarma and Gangele (2014)	$\hat{\bar{Y}}_{VS}=\bar{y}\ exp\left[\left\{\frac{\bar{x}'(\alpha\bar{Z}+\beta)}{(\alpha\bar{Z}'+\beta)}-\bar{x}\right\}\Big/\left\{\frac{\bar{x}'(\alpha\bar{Z}+\beta)}{(\alpha\bar{Z}'+\beta)}+\bar{x}\right\}\right]$	1	α	β
Vishwakarma and Kumar (2014)	$\bar{y}^{dc}_{RPe}=\bar{y}\left[\begin{array}{l}k\ exp\left\{\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z}-\bar{x}\right)\Big/\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z}+\bar{x}\right)\right\}\\+(1-k)\ exp\left\{\left(\bar{x}-\frac{\bar{x}'}{\bar{z}'}\bar{Z}\right)\Big/\left(\bar{x}+\frac{\bar{x}'}{\bar{z}'}\bar{Z}\right)\right\}\end{array}\right]$	k	1	0

To the first degree of approximation, the MSE expressions of various estimators are

$$MSE\left(\hat{\bar{Y}}_{ReMd}\right)=\bar{Y}^2\left\{f_1C_y^2+f_3\left(\frac{1}{4}C_x^2-\rho_{yx}C_yC_x\right)\right\} \tag{3.1}$$

$$MSE\left(\hat{\bar{Y}}_{PeMd}\right)=\bar{Y}^2\left\{f_1C_y^2+f_3\left(\frac{1}{4}C_x^2+\rho_{yx}C_yC_x\right)\right\} \tag{3.2}$$

$$MSE\left(\hat{\bar{Y}}_{\mathrm{Re}}^{dc}\right)=\bar{Y}^2\left\{f_1C_y^2+\frac{1}{4}\left(f_3C_x^2+f_2C_z^2\right)-\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_Z\right)\right\} \tag{3.3}$$

$$MSE\left(\hat{\bar{Y}}_{Pe}^{dc}\right)=\bar{Y}^2\left\{f_1C_y^2+\frac{1}{4}\left(f_3C_x^2+f_2C_z^2\right)+\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_Z\right)\right\} \tag{3.4}$$

$$MSE\left(\hat{\bar{Y}}_{vs}\right)=\bar{Y}^2\left\{f_1C_y^2+\frac{1}{4}\left(f_3C_x^2+\lambda_1^2f_2C_z^2\right)-\left(f_3\rho_{yx}C_yC_x+\lambda_1f_2\rho_{yz}C_yC_Z\right)\right\} \tag{3.5}$$

$$\begin{aligned}MSE\left(\bar{y}_{RPe}^{dc}\right)=\bar{Y}^2\Bigg\{&f_1C_y^2+\frac{(2k-1)^2}{4}\left(f_3C_x^2+f_2C_z^2\right)\\&-(2k-1)\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_Z\right)\Bigg\}\end{aligned} \tag{3.6}$$

Also, the minimum attainable MSE of $\hat{\bar{Y}}_{VS}$ and $\bar{y}_{RPe}^{dc}$ care given, respectively, by

$$MSE\left(\hat{\bar{Y}}_{VS}\right)_{min}=\bar{Y}^2\left\{f_1C_y^2+\frac{1}{4}f_3\left(C_x^2-4\rho_{yx}C_yC_x\right)-f_2\rho_{yz}^2C_y^2\right\} \tag{3.7}$$

$$MSE\left(\bar{y}_{RPe}^{dc}\right)_{min}=\bar{Y}^2\left\{f_1C_y^2-\frac{\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_z\right)^2}{f_3C_x^2+f_2C_z^2}\right\} \tag{3.8}$$

4. Efficiency Comparisons

The variance of the usual unbiased estimator $\bar{y}$ for the population mean $\bar{Y}$ in simple random sampling without replacement (SRSWOR) is

$$V\left(\bar{y}\right)=f_1S_y^2=f_1\bar{Y}^2C_y^2 \tag{4.1}$$

Also, the MSE of the chain ratio estimator $\overline{y}_R^{(C)}$ in (1.1) is given by

$$MSE\left\{\overline{y}_R^{(C)}\right\} = \overline{Y}^2\left(f_1C_y^2 + f_3C_x^2 + f_2C_z^2 - 2f_3\rho_{yx}C_yC_x - 2f_2\rho_{yz}C_yC_z\right) \quad (4.2)$$

For making efficiency comparisons of the class T with the existing estimators, we have from (2.7), (3.1) to (3.6), (4.1) and (4.2),

(i) $MSE(T) < V(\overline{y})$ if

$$C_y > \frac{1}{4}\left[\frac{(2k-1)\left(f_3C_x^2 + \lambda_1^2 f_2C_z^2\right)}{f_3\rho_{yx}C_x + \lambda_1 f_2\rho_{yz}C_z}\right] \quad (4.3)$$

(ii) $MSE(T) < MSE\left\{\overline{y}_R^{(C)}\right\}$ if

$$C_y < \frac{1}{4}\left[\frac{(2k+1)(3-2k)f_3C_x^2 - \left\{(2k-1)^2\lambda_1^2 - 4\right\}f_2C_z^2}{(3-2k)f_3\rho_{yx}C_x - \left\{(2k-1)\lambda_1 - 2\right\}f_2\rho_{yz}C_z}\right] \quad (4.4)$$

(iii) $MSE(T) < MSE\left(\hat{\hat{\overline{Y}}}_{ReMd}\right)$ if

$$C_y < \frac{1}{4}\left[\frac{4k(1-k)f_3C_x^2 - (2k-1)^2\lambda_1^2 f_2C_z^2}{2(1-k)f_3\rho_{yx}C_x - (2k-1)\lambda_1 f_2\rho_{yz}C_z}\right] \quad (4.5)$$

(iv) $MSE(T) < MSE\left(\hat{\hat{\overline{Y}}}_{PeMd}\right)$ if

$$C_y > -\frac{1}{4}\left[\frac{4k(1-k)f_3C_x^2 - (2k-1)^2\lambda_1^2 f_2C_z^2}{2kf_3\rho_{yx}C_x + (2k-1)\lambda_1 f_2\rho_{yz}C_z}\right] \quad (4.6)$$

(v) $MSE(T) < MSE\left(\hat{\hat{\overline{Y}}}_{Re}^{dc}\right)$ if

$$C_y < \frac{1}{4}\left[\frac{4k(1-k)f_3C_x^2 - \left\{(2k-1)^2\lambda_1^2 - 1\right\}f_2C_z^2}{2(1-k)f_3\rho_{yx}C_x - \left\{(2k-1)\lambda_1 - 1\right\}f_2\rho_{yz}C_z}\right] \quad (4.7)$$

(vi) $MSE(T) < MSE\left(\hat{\bar{Y}}_{Pe}^{dc}\right)$ if

$$C_y > -\frac{1}{4}\left[\frac{4k(1-k)f_3C_x^2 - \left\{(2k-1)^2\lambda_1^2 - 1\right\}f_2C_z^2}{2kf_3\rho_{yx}C_x + \left\{(2k-1)\lambda_1 + 1\right\}f_2\rho_{yz}C_z}\right] \tag{4.8}$$

(vii) $MSE(T) < MSE\left(\hat{\bar{Y}}_{VS}\right)$ if

$$C_y < \frac{k}{2}\left(\frac{f_3C_x^2 + \lambda_1^2 f_2C_z^2}{f_3\rho_{yx}C_x + \lambda_1 f_2\rho_{yz}C_z}\right) \tag{4.9}$$

(viii) $MSE(T) < MSE\left(\bar{y}_{RPe}^{dc}\right)$ if

$$C_y > \frac{1}{4}\left\{\frac{(2k-1)(\lambda_1+1)f_2C_z^2}{f_2\rho_{yz}C_z}\right\} \tag{4.10}$$

5. Empirical Study

To examine the merits of the proposed class *T*, we have considered four natural population data sets and the *percentage relative efficiencies* (PREs) are obtained for various suggested estimators of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$. The findings are presented in Table 2. The description of the populations and the required values of the various parameters are given below:

Population I - [Source: Anderson (1958)]

Y: Head length of second son

X : Head length of first son

Z : Head breadth of first son

N = 25, n' = 10, n = 7, $\bar{Y}$ = 183.84, $\bar{X}$ = 185.72, $\bar{Z}$ = 151.12,

ρ_{yx} = 0.7108, ρ_{yz} = 0.6932, ρ_{xz} = 0.7346, C_y = 0.0546, C_x = 0.0526,

C_z = 0.0488

Population II - [Source: Cochran (1977)]

Y: Number of ‘placebo’ children

X: Number of paralytic polio cases in the ‘placebo’ group

Z: Number of paralytic polio cases in the ‘not inoculated’ group

N = 34, n' = 15, n = 10, $\bar{Y}$ = 4.92, $\bar{X}$ = 2.59, $\bar{Z}$ = 2.91,

ρ_{yx} = 0.7326, ρ_{yz} = 0.6430, ρ_{xz} = 0.6837, C_y^2 = 1.0248, C_x^2 = 1.5175,

C_z^2 = 1.1492

Population III - [Source: Murthy (1967)]

Y : Area under wheat in 1964

X: Area under wheat in 1963

Z: Cultivated area in 1961

$N = 34, n' = 10, n = 7, \bar{Y} = 199.44, \bar{X} = 208.89, \bar{Z} = 747.59,$

$\rho_{yx} = 0.9801, \rho_{yz} = 0.9043, \rho_{xz} = 0.9097, C_y^2 = 0.5673, C_x^2 = 0.5191,$

$C_z^2 = 0.3527$

Population IV - [Source: Sukhatme and Chand (1977)]

Y : Apple trees of bearing age in 1964

X: Bushels of apples harvested in 1964

Z: Bushels of apples harvested in 1959

$N = 200, n' = 30, n = 20, \bar{Y} = 1031.82, \bar{X} = 2934.58, \bar{Z} = 3651.49,$

$\rho_{yx} = 0.93, \rho_{yz} = 0.77, \rho_{xz} = 0.84, C_y^2 = 2.55280, C_x^2 = 4.02504,$

$C_z^2 = 2.09379$

6. Conclusion

It is observed from Table 2 that, for all the population data sets, the PRE of the proposed class T is greater than the usual unbiased estimator $\bar{y}$ and the chain ratio estimator $\bar{y}_R^{(C)}$. Also, the members of the class T, that have been considered in Section 3 and are listed in Table 2, does not attain the optimum values of the PRE in all the data sets except for the Vishwakarma and Kumar (2014) estimator $\bar{y}_{RPe}^{dc}$ in populations I and II, in which case the PRE is almost optimum. Therefore, the proposed class T is more appropriate for estimating the population mean $\bar{Y}$ of the study variate Y. Hence, the use of the proposed class T should be preferred over other estimators.

Table 2: Percentage Relative Efficiencies (PREs) of Different Estimators of $\overline{Y}$ with Respect to $\overline{y}$

Estimators	Population I	Population II	Population III	Population IV
$\overline{y}$	100	100	100	100
$\overline{y}_R^{(C)}$	178.82	136.91	730.81	279.93
$\hat{\overline{Y}}_{ReMd}$	123.25	132.66	136.57	140.16
$\hat{\overline{Y}}_{PeMd}$	*	*	*	*
$\hat{\overline{Y}}_{Re}^{dc}$	176.54	184.36	259.55	247.82
$\hat{\overline{Y}}_{Pe}^{dc}$	*	*	*	*
$\hat{\overline{Y}}_{VS}$	188.31	186.70	447.61	293.97
$\overline{y}_{RPe}^{dc}$	196.28	189.27	763.30	322.95
T	196.39	189.27	779.54	326.41

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REFERENCES

ANDERSON, T. W., 1958, *An Introduction to Multivariate Statistical Analysis*, New York: John Wiley and Sons.

CHAND, L.,1975, Some ratio type estimators based on two or more auxiliary variables, Ph.D. dissertation, Iowa State University, Ames, Iowa (USA).

COCHRAN, W. G.,1940, The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce, *The Journal of Agricultural Science* 30 (02): 262-275.

COCHRAN, W. G., 1977, *Sampling Techniques*, New York: John Wiley and Sons.

FULLER, W. A., 2000, Two-phase sampling, SSC Annual Meeting Proceeding of the Survey Methods Section, 23-30.

HIDIROGLOU, M. A., 2001, Double sampling, *Survey Methodology* 27 (2): 143-154.

HIDIROGLOU, M. A. AND SARNDAL, C. E.,1998, Use of auxiliary information for Two-phase sampling, *Survey Methodology* 24 (1): 11-20.

KIREGYERA, B., 1980, A chain ratio-type estimator in finite population double sampling using two auxiliary variables, *Metrika* 27 (1): 217-223.

MURTHY, M. N., 1964, Product method of estimation. *Sankhyā: The Indian Journal of Statistics* Series A, 26 (1): 69-74.

MURTHY, M. N., 1967, *Sampling Theory and Methods*, Calcutta, India: Statistical Publishing Soc.

NEYMAN, J., 1938, Contribution to the theory of sampling human populations, *Journal of the American Statistical Association* 33 (201): 101-116.

PRASAD, B., SINGH, R. S. AND SINGH, H. P.,1996, Some chain ratio-type estimators for ratio of two population means using two auxiliary characters in two phase sampling, *Metron* LIV: 95-113.

ROBSON, D. S., 1957, Applications of multivariate polykays to the theory of unbiased ratio-type estimation, *Journal of the American Statistical Association* 52 (280): 511-522.

SINGH, B. K. AND CHOUDHURY, S.,2012, Exponential chain ratio and product type estimators for finite population mean under double sampling scheme, *Global Journal of Science Frontier Research* 12 (6): 12-24.

SINGH, G. N. AND UPADHYAYA, L. N., 1995, A class of modified chain-type estimators using two auxiliary variables in two phase sampling, *Metron* LIII: 117-125.

SINGH, H. P. AND VISHWAKARMA, G. K., 2007, Modified exponential ratio and product estimators for finite population mean in double sampling, *Austrian Journal of Statistics*, 36 (3): 217-225.

SUKHATME, B. V. AND CHAND, L., 1977, Multivariate ratio-type estimators, Proceedings of American Statistical Association, Social Statistics Section, 927-931.

VISHWAKARMA, G. K. AND GANGELE, R. K., 2014, A class of chain ratio-type exponential estimators in double sampling using two auxiliary variates, *Applied Mathematics and Computation* 227: 171-175.

VISHWAKARMA, G. K. AND KUMAR, M., 2014, An improved class of chain ratio-product type estimators in two-phase sampling using two auxiliary variables, *Journal of Probability and Statistics* 2014: 1-6.