Value-at-Risk Measures for the PSE Index Using Hidden Markov Models

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Value-at-Risk (VaR) that measures the PSE index are estimated using an *m*-state normal-hidden Markov model. The estimation procedure will be done under unconditional and conditional approaches. Backtesting will be done to assess how well the estimates performed.

Keywords: homogeneous, irreducible, aperiodic Markov Chain

1. Introduction

A Hidden Markov Model (HMM) is a statistical model of a sequence of observed random variables $\{X_t\}_{t=1}^{\infty}$ whose distribution depends on another sequence of underlying unobserved random variables $\{C_t\}_{t=1}^{\infty}$, the latter being governed by a Markov process.More formally, we say that a HMM is characterized by the following relationship:

$$\Pr(C_t | C_{t-1}, C_{t-2}, \ldots, C_1) = \Pr(C_t | C_{t-1}), \ t = 2, 3, \ldots \tag{1}$$

$$\Pr(X_t | X_{t-1}, X_{t-2}, \ldots, X_1, C_t, C_{t-1}, \ldots, C_1) = \Pr(X_t | C_t), \ t = 1, 2, 3, \ldots \tag{2}$$

That is, the unobserved "state-space" process $\{C_t: t = 1, 2, 3, \ldots\}$ satisfy the Markov property and the observed "state-dependent" process $\{X_t\}$ is conditionally independent of its past given the concurrent state C_t. If C_t assumes a finite number of values $\{1, 2, \ldots, m\}$, then we call it an *m-state HMM.*

Applications of HMMs can be found in fields as diverse as biophysics (ion channel modeling), earth and environmental sciences (wind direction, climate change), temporal pattern recognition (facial, speech, handwriting, gait, gesture, etc.), engineering (speech and signal processing), bioinformatics (biological sequencing), among others.In the financial/econometrics literature, HMMs can be classified under the more general heading of Markov regime-switching (MS) models, wherein the m states of the HMM correspond to the m regimes of the latter class of models. Among the different extensions of the MR models, HMMs exhibits a very simple form, wherein, for each regime, only a distribution function is assumed. More compound forms of MR models have found their way into the literature. Notable among these are MR state-space models (Kim & Nelson, 1999), MR ARCH (Cai, 1994), MR regression (see references on Frühwirth-Schnatter, 2006), and MR GARCH (Marcucci, 2005). Multivariate versions are also available (Billio & Pelizzon, 2000). However, despite the simplicity exhibited by HMMs, they still remain as viable choices for model development.

The term HMM actually refers to a class of models. This paper deals mostly with what Zucchini and MacDonald (2009) refer to as the *basic HMM*, that is, univariate observations with neither trend nor seasonal variation and that the unobserved process is that of a homogenous first-order Markov chain. In the MR context, we could think of this basic HMM as a MR distribution model, i.e. in each regime, we model X_t by a probability distribution (either by a probability density function or a probability mass function depending on whether the observed sequence is continuous or discrete). A further simplification that is adopted by this paper is to assume that the distribution in each regime is that of the Gaussian (or normal) distribution. Though it is widely accepted that asset returns are not normally distributed, a mixture of Gaussians that results from HMM models do exhibit the skewness and leptokurtic characteristics (high peaks and fat tails) of such returns. Instead of dealing with *returns*, however, the rest of this paper will deal with *loss* (which is nothing more than *negative returns.*)

There is another reason for considering HMMs. Even if the aim is to develop a more complex MR model, results from HMM analyses provide insights on the dynamics of the observed process that would have gone unnoticed had it just proceeded with a pre-determined number of states (regimes). Aside from modeling the PSE index using HMM, the aim of this paper is to exhibit estimates of risk measures which is an offshoot of such a modeling procedure. Section 2 provides some background on HMMs and defining the VaR given the HMM, while section 3 provides results of analyses.

2. Hidden Markov Model and Risk Measures

Let us denote the observed process by $\{X_t\}_{t=1}^{\infty}$ and the unobserved process by $\{C_t\}_{t=1}^{\infty}$. Additionally, let us assume that the $\{C_t\}$ is an irreducible, homogeneous and aperiodic Markov process and that it takes values from a finite set of m states $\{1, 2,\ldots, m\}$. Following the notations of Zucchini and MacDonald (2009) we shall denote

- the past history of $\{C_t\}$ up to time i by $C^{(i)} = C_1, C_2, \ldots. C_i$;
- the *transition probability matrix* by $\Gamma = (\gamma_{jk})$, i.e.,$\gamma_{jk} = \Pr\{C_i = k \mid C_{i-1} = j\}$;

- the *unconditional probabilities* of the Markov chain (MC) being in a given state at a given time t by the row vector $u(t) = (\Pr(C_t = 1), \Pr(C_t = 2), \ldots, \Pr(C_t = m))$;
- the *initial distribution* of the MC is denoted by δ, i.e., $\delta = u(0)$;
- the past history of $\{X_t\}$ up to time i by $X^{(i)} = X_1, X_2, \ldots X_i$;
- the state-dependent distribution by $p_i(x) = Pr(X_t = x | C_t = i)$, $i = 1, 2, \ldots, m$; (Here, p_i represents the probability mass function (pmf) of X_t for discrete observations whereas it represents the probability density function (pdf) for continuous observations.)
- the m x m diagonal matrix of state-dependent distributions by $P(x) = \text{diag}(p_1(x), \ldots, p_m(x))$;
- the vector of *forward probabilities at a given time t* by $\alpha_t = (\alpha_t(1), \ldots, \alpha_t(m))$ where $\alpha_t(j) = \Pr(X^{(t)} = x^{(t)}, C_t = j)$, $j = 1, 2, \ldots, m$;
- the vector of *backward probabilities at a given time t* by $\beta_t = (\beta_t(1), \ldots, \beta_t(m))$ where $\beta_t(j) = \Pr(X_{t+1}^T = x_{t=1}^T | C_t = j)$, $j = 1, 2, \ldots, m$.
- The likelihood $lik_T = \Pr(C^{(T)}, X^{(T)}) = \Pr(C_1) \Pr(X_1|C_1) \prod_{k=2}^{T} \Pr(C_k | C_{k=1}) \Pr(X_k | C_k)$ can now be expressed in matrix form as $lik_T = \delta' P(x_1)\, \Gamma P(x_2) \ldots \Gamma P(x_T)\, 1_m$.

We now consider the two most common risk measures: the Value-at-Risk (VaR) and expected shortfall (ES). In computing these risk measures, two different approaches present themselves: the unconditional (or static) approach and the conditional (or dynamic) approach. In the unconditional approach, we treat the observed losses as values generated by some distribution F while in the conditional approach we model the observed losses as a sequence from a time-indexed process. This paper will limit itself to developing VaR based on HMMs.

VaR measures the largest potential loss one can incur in a given time horizon at a given confidence level. This simple definition consists of three components: (1) a probability distribution, which could either be a density function f or a cumulative distribution function F, for the random variable L which represents *loss* ; (2) a time horizon Δ, usually $\Delta = 1$ day or $\Delta = 10$ days; and (3) a confidence level $(1- \alpha)$, typically 95% or 99%, that is, α is either 0.05 or 0.01. For this reason, VaR is often represented as $VaR_{1-\alpha}^{\Delta}$. More formally, if F_L denotes the cumulative distribution function (*cdf*) of the random variable (*rv*), L representing *loss incurred in the given time horizon* Δ, then

$$VaR_{1-\alpha}^{\Delta} = \inf\{l \in \Re : F_L(l) \geq 1 - \alpha\} \tag{3}$$

i.e, VaR is simply the $100(1-\alpha)$th quantile of the loss cdf. As for the conditional approach, the VaR would be the $100(1-\alpha)$th quantile of the distribution of the predicted value $\hat{X}_{T+\Delta}$.

The basis for coming up with an unconditional approach using a HMM lies in the result that given a homogeneous, irreducible and aperiodic MC, a limiting distribution δ* = diag$\left(lim_{t\to\infty}\Gamma^{t}\right)$ exists. Essentially, this tells us that the long run distribution of the generated loss values may be expressed by the mixture distribution f(x) = $\sum_{i=1}^{m}\delta_i^{*}\, p_i(x)$.

Assuming that the $X_t|C_t=i \sim N(\mu_i\,,\sigma_i^2)$, then the estimated VaR is the 100 (1-a)th quantile of the mixture f(x) =$\sum_{i=1}^{m}\delta_i^{*}\phi_i(x;\mu_i,\sigma_i^2)$, that is, the 100(1-$a$)th quantile of a convex combination of normal pdfs.

Numerical methods have to be employed to derive the quantile for this mixture distribution since it is not simply the convex combination of the corresponding quantiles.

On the other hand, estimates for VaR under the conditional approach uses the state prediction formula for HMM given by

$$P(C_{T+\Delta}=i\,|\,X^{(T)})=\frac{\alpha_T^{'}\Gamma^{\Delta}(,i)}{1'\alpha_T}\text{, i = 1, 2,…,m} \tag{4}$$

where $\Gamma^{\Delta}(, i)$ is the i^{th} column of the matrix Γ^{Δ}. Given the respective probabilities that the system is in regime i at time $t = T+\Delta$, then the VaR is the quantile of the normal distribution corresponding to the state (or regime) that the system is most likely at time $T + \Delta$. In symbols,

$$VaR_{1-\alpha}^{\Delta}=\mu_{i^*}+\sigma_{i^*}\Phi^{-1}\left(1-\alpha\right), \tag{5}$$

where

$$\text{i*} = \arg\max_{i=1,2,\dots,m}\left\{P(C_{T+\Delta}=i\,|\,X^{(T)})\right\}. \tag{6}$$

For the conditional approach, we modify equations (1) and (3), as

$$P\left(C_t=i\,|\,X^{(T)}\right)=\frac{\alpha_t(i)\beta_t(i)}{lik_T} \tag{4`}$$

and with

$$i^* = \arg\max_{i=1,2,\dots,m}\left\{P(C_t=i\,|\,X^{(T)})\right\}\text{, respectively.} \tag{6`}$$

We use equation (5) to compute for VaR_t for each time period t.

To assess the performance of the VaR estimates, several tests will be performed, a number of which are based on the indicator notation

$$\hat{I}_{t+1,\alpha} \equiv I\left(L_{t+1} > Va\hat{R}^{t}_{1-\alpha}\right) \tag{7}$$

and the statistic $$\hat{\pi}_{\alpha} = \frac{\sum_{k=1}^{T} \hat{I}_{k,\alpha}}{T} = \frac{n^*}{T}. \tag{8}$$

The time of first failure (TUFF) test is based on the number of observations before the first VaR violation. Under the null hypothesis H_0: $\alpha = \alpha^*$, the likelihood ratio test is

$$LR_{TUFF} = 2log\left[\frac{\left(\frac{1}{t^*}\right)\left(1-\frac{1}{t^*}\right)^{t^*-1}}{\hat{\pi}_{\alpha^*}\left(1-\hat{\pi}_{\alpha^*}\right)^{t^*-1}}\right] \overset{asymp}{\sim} \chi^2_1, \tag{9}$$

where t^* denotes time until the first exception.

Kupiec (1995) noted that the TUFF test has limited power to distinguish among alternative hypotheses because all observations after the first failure are ignored and proposed a nonparametric test based on the proportion of exceptions. Under the null hypothes is, H_0: $\alpha = \alpha^*$, the likelihood ratio test (also known as the proportion of failure test and also as the unconditional coverage test) is

$$LR_{POF} = LR_{UC} = 2log\left[\frac{\left(1-\hat{\pi}_{\alpha^*}\right)^{T-n^*}\hat{\pi}_{\alpha^*}^{n^*}}{\left(1-\alpha^*\right)^{T-n^*}\alpha^{*n^*}}\right] \overset{asymp}{\sim} \chi^2_1. \tag{10}$$

Christoffersen (1983) argued that Kupiec's LR_{UC} test does not give any information about the temporal dependence of violations and ignores conditioning coverage, since violations could cluster over time. A test for independence over time of VaR violations is given by

$$LR_{IND} = 2\log\left[\frac{\left(1-\hat{\pi}_0\right)^{n_{00}}\hat{\pi}_0^{\,n_{01}}\left(1-\hat{\pi}_1\right)^{n_{10}}\hat{\pi}_1^{\,n_{11}}}{\left(1-\hat{\pi}_2\right)^{n_{00}+n_{10}}\hat{\pi}_2^{\,n_{01}+n_{11}}}\right] \overset{asymp}{\sim} \chi^2_1, \tag{11}$$

where

$n_{00} \equiv$ number of two consecutive time periods with no violation,

$n_{10} \equiv$ number of time periods with no violation preceded by a time period with a violation,

n_{01} ≡ number of time periods with a violation preceded by a time period with no violation,

n_{11} ≡ number of two consecutive time periods of with violations,

$\hat{\pi}_0 = n_{01} / (n_{01} + n_{00})$,

$\hat{\pi}_1 = n_{11} / (n_{11} + n_{10})$, and

$\hat{\pi}_2 = (n_{01} + n_{11}) / (n_{01} + n_{00} + n_{11} + n_{10})$.

In addition, Christoffersen (1998) also proposed a test of correct conditional coverage which tests the null hypothesis of an independent failure process with probability α* against the alternative of a first order Markov failure process, given by

$$\mathrm{LR}_{CC} = 2\log\left[\frac{(1-\hat{\pi}_0)^{n_{00}}\,\hat{\pi}_0^{\,n_{01}}\,(1-\hat{\pi}_1)^{n_{10}}\,\hat{\pi}_1^{\,n_{11}}}{(1-\alpha^*)^{T-n^*}\,\alpha^{*n^*}}\right] \overset{asymp}{\sim} \chi_2^2 . \tag{12}$$

Finally, the proposed conditional VaR will also be subjected to the dynamic conditional quantile (DQ) test of Engle and Manganelli (2004) using the following statistic

$$\mathrm{DQ} = \left(h_t' X_t \left[X_t' X_t\right]^{-1} X_t' h_t\right) / T\alpha^*(1-\alpha^*), \tag{13}$$

where $h_t = \hat{I}_{t,\alpha^*} - \alpha^*$ is the demeaned violations, and X_t is a vector of instruments which might include lags of h_t, VaR_t and its lags.

Under the null hypothesis that h_t and X_t are orthogonal, the proposed DQ statistic follows a χ_q^2 distribution with $q = \text{rank}\, X_t$. Using Monte-carlo experiments, Berkowitz et al. (2011) showed that the DQ test with $X_t = VaR_t$ appears to be the best backtest for 1% VaR models, and other backtests generally have much lower power against misspecified VaR models.

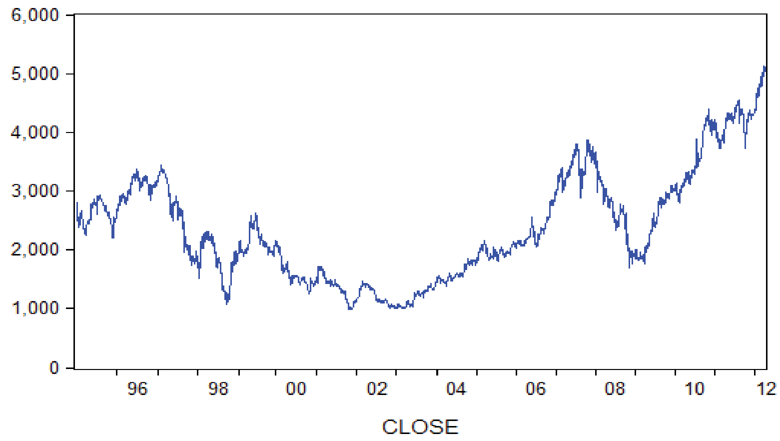
3. Results and Discussion

Our data consist of values at the close of trading day of the Philippine Stock Exchange index (PSEi) covering the period from 3 January 1995 to 30 March 2012. Losses were computed by taking the negative difference of log values and expressed in percentage, i.e.

$$Loss_t = -log\left(\frac{PSEi_t}{PSEi_{t-1}}\right) \times 100\% . \tag{14}$$

Figure 1 shows the graphs of the PSEi and the corresponding loss values for the period covered.

Figure 1 **PSEi (CLOSE) from January 3, 1995 to March 30, 2012 and Loss from January 4, 1995 to March 30, 2012.**



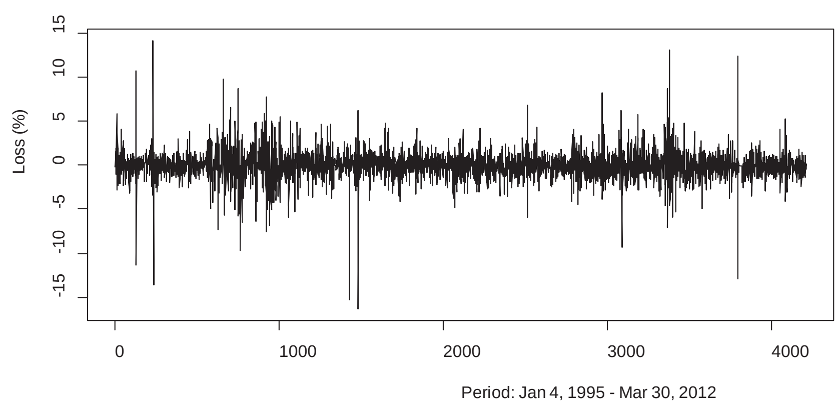
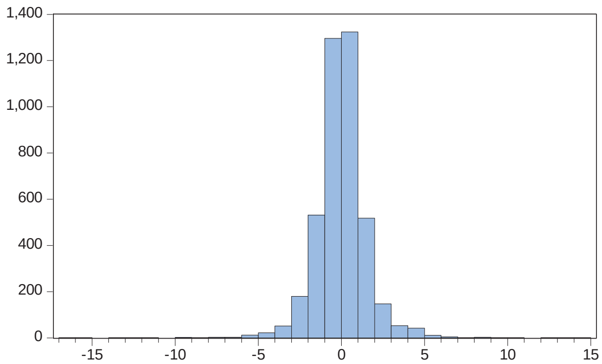


Figure 2 shows a histogram of the percentage losses together with some descriptives. Noteworthy are the values for the skewness and kurtosis statistics which suggests a departure from normality. The *p*-value given by the Jarque-Bera test statistic for normality clearly rejects the null hypothesis that losses are normally distributed.

Figure 2 Histogram of Loss together with some descriptives



Series: LOSS(x100%)	
Sample 1/03/1995 3/30/2012	
Observations 4212	
Mean	-0.014041
Median	0.000000
Maximum	14.21005
Minimum	-16.37602
Std. Dev.	1.589490
Skewness	-0.247755
Kurtosis	17.08539

Results of fitting a number of m states, $m = 2, 3, 4, 5$ are presented in Table 1. All three information criteria suggest the best fit among those considered would be a 4-state HMM.

Table 1 Values of Akaike's Information Criterion (AIC), corrected AIC (AICc) and Schwartz' Bayesian Information Criterion (BIC)[4] for 2-state to 5-state HMM

	No. of parameters (p)	AIC[1]	AICc[2]	BIC[3]
2-state HMM	8	-24366.300	-24366.223	-24346.806
3-state HMM	21	-24572.820	-24572.600	-24538.706
4-state HMM	32	-24630.840	-24630.335	-24578.856
5-state HMM	45	-24626.40	-24625.406	-24553.298

The Baum-Welch algorithm (a variant of the EM algorithm) gives the values of the estimates of the parameters for the 4-state HMM.

Stationary Probabilities $(.0542 \quad .2045 \quad .2331 \quad .5082)$

Mean $(0.020 \quad 0.822 \quad -0.846 \quad 0.026)$

Standard Deviation $(4.496 \quad 1.451 \quad 1.260 \quad 0.790)$

Transition Probabilities

$$\begin{pmatrix} 0.812 & 0.087 & .0101 & 0 \\ 0.050 & 0.747 & 0.203 & 0 \\ 0 & 0.137 & 0.722 & 0.141 \\ 0.001 & 0.030 & 0.034 & 0.935 \end{pmatrix}$$

Initial Distribution $(0 \quad 1 \quad 0 \quad 0)$

The resulting normal-mixture density for the loss x is given by

$$f(x) = .0542\ \phi(x; m = 0.020,\ \sigma = 4.496) + .2045\ \phi(x; m = 0.822, \sigma = 1.451) + .2331\phi(x; m = -0.846,\ \sigma = 1.26) + \ .5082\ \phi(x; m = 0.026,\ \sigma = 0.79)).$$

Figure 3 shows the four normal density functions (weighted in accordance with the stationary probabilities) that make up fitted mixed distribution.

Figure 3 Loss Histogram and the Probability Densities that Make Up Fitted Mixed Distribution

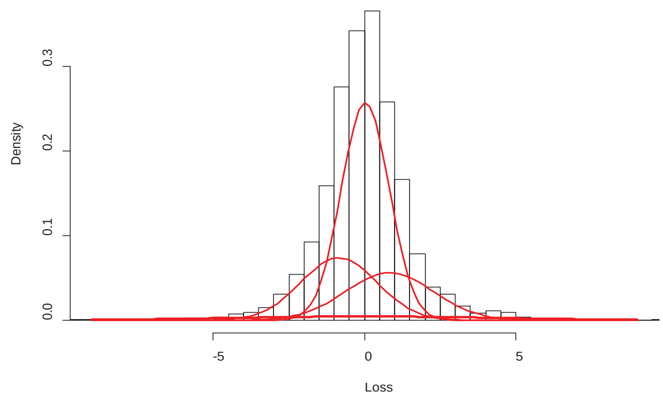


Figure 4 can help us interpret the four different regimes suggested by the HMM. The first state (though few in terms of representation) refers to those times where volatility is extremely high. It could be noticed that all the very high/low spikes are classified as being in this state. The second state is that of a general downward trend in the behavior of original series (about 20% of the time, as reflected in the vector of stationary probabilities), while the third state is the opposite, that is, a general upward trend. However, most of the time (about 51%), the system is behaving in a stable manner.

Figure 4 Classification of Observed Values According to the Most Likely State at Time *t*.

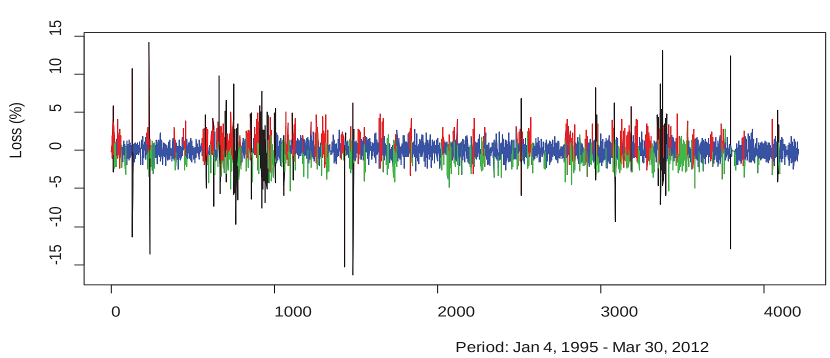


Table 4 shows the results of the various backtesting procedures for the unconditional VaR. The TUFF and UC tests seem to indicate that there is no sufficient evidence that the probabilities of VaR violations are not equal to α at .05 level of significance.

Table 4 ***p*-values for the Different Backtesting Procedures**

α	**TUFF**	**UC**	**IND**	**CC**
.05	.3512	.9662	< 0.000	< 0.000
.01	.0611	.7726	< 0.000	< 0.000

The results for the IND and CC tests, however, seem to indicate that the proposed unconditional VaR cannot take into account the clustering of VaR violations. This is to be expected since the unconditional VaR (being a fixed quantity) is not designed to address the temporal fluctuations of the losses. Rather it is intended to give a value such that, in the long run, losses exceeding this value would only occur 100 α% of the time. To give a proper perspective to the unconditional VaR derived from the 4-state HMM, Table 5 gives the VaR estimates derived from several members of the generalized hyperbolic distribution for 1-α = 0.95 and 0.99.

Table 5 **Estimated VaR for the 4-state HMM and selected members from the family of generalized hyperbolic distribution, namely the asymmetric *t*, the asymmetric normal inverse Gaussian,the asymmetric variance gamma, and the generalized hyperbolic distributions. See (McNeil et al, 2005) for a description of these distributions**

1-α	**4-state HMM**	**T**	**NIG**	**VG**	**GHyp**
.95	2.313518	2.254813	2.458267	2.391257	2.25925
.99	4.462215	4.328036	4.351832	4.02871	4.33931

Results shown in Table 6, the proposed conditional VaR, at .05 level of significance, underestimated $VaR_{.01}$, as shown by the UC and CC columns. A 95% confidence interval for the proportion of violations gives (.0101, .0175), which seems to translate to a difference of 1 to 75 basis points. Other than that, results show that VaR estimates from HMMs could present a viable option for VaR modeling.

Table 6 *p*-values for the different backtesting procedures applied to the conditional VaR estimate

α	TUFF	UC	IND	CC	DQ
.05	.6458	.6525	.2502	.4424	.9999
.01	.5494	.0287	.2355	.0447	.9693

4. Conclusion

Despite its simplicity, HMMs perform fairly well in modeling processes with varying system-behavior. Even the goal is to fit a more complicated Markov switching regime model, it would do well to use HMMs as an exploratory tool. Proper modeling involves using the right number of states (or regimes) that should be based on the data at hand and not on the basis of one's subjective motives or perceptions.

NOTES

1. AIC = $2p$ – 2 log-likelihood
2. Corrected AIC: AICc = AIC + $\frac{2p(p+1)}{T-p-1}$
3. BIC = p log(T) – 2 log-likelihood
4. Some authors give a different formula for the AIC and BIC by a multiple of 1/T.

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