Length of a Time Series for Seasonal Adjustment: Some Empirical Experiments*

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Use of 5 to 15 years of quarterly or monthly data is suggested when doing seasonal adjustment using X11 and its variants. This is meant to address changes in the structure of the time series. Philippine time series are good candidates for this practice since they usually exhibit frequent changes in patterns. Empirical validation of the suggested length of series is done for seasonal ARMA processes. Different quarterly series were simulated for the following situations and seasonal adjustment was done for various lengths of time series: (1) processes without any structural change; (2) processes with abrupt permanent change in structure; (3) processes with gradual permanent change in structure. For all types of processes, both weak and strong seasonality were considered. Regression models were used in testing the effect of length of series used in seasonal adjustment to the error in estimating the seasonal factor. Results show that the length of series used does not have significant effect on the seasonal adjustment for processes without structural change and with abrupt permanent structural change. On the other hand, for processes with gradual permanent change, use of longer lengths of series for seasonal adjustment is better.

Keywords: seasonal adjustment, seasonal factor, X11-ARIMA, seasonal ARMA processes

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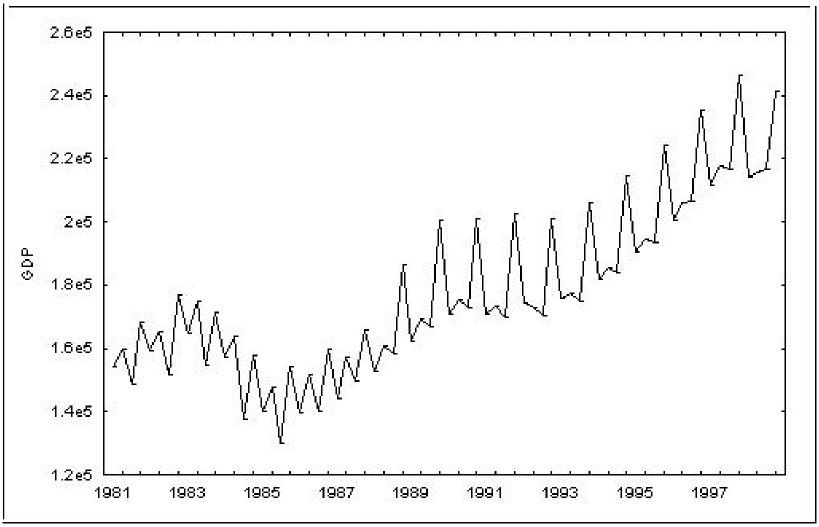
1. Introduction

Seasonal adjustment of series with changes in behavior usually results in a seasonally adjusted series that is not smoothened. Such situations may lead to misleading analyses (Ghysels, 1988). Specific illustrations of such situations are discussed by Castro and Osborn (2004) for periodic autoregressive processes whose periodicity remains, though in an altered form, after X11 seasonal adjustment. Thus, tests to detect deviations from deterministic seasonal patterns were developed (e.g.,Canova and Hansen, 1995; Busetti and Harvey, 2003). Of interest, however, is how series length impacts on the seasonal adjustment under non-deterministic seasonality. This issue of series length is a concern addressed by Findley and Martin (2003) who studied the performance of TRAMO SEATS and X11-ARIMA for short and moderate length series.

In the Philippines where official seasonal adjustment is done using X11-ARIMA, the problem of seasonally adjusting series with changing behavior is addressed by limiting the length of the series for seasonal adjustment to the suggested length of 5 to 15 years (Dagum, 1988; Bersales and Sarte, 1999). This practice has been done by agencies doing official seasonal adjustment since Philippine time series usually exhibit frequent changes in patterns. For example, quarterly Philippine Gross Domestic Product (GDP) from 1981 to 1998 as presented in Figure 1 shows changes in pattern with the new seasonal pattern starting in 1988.

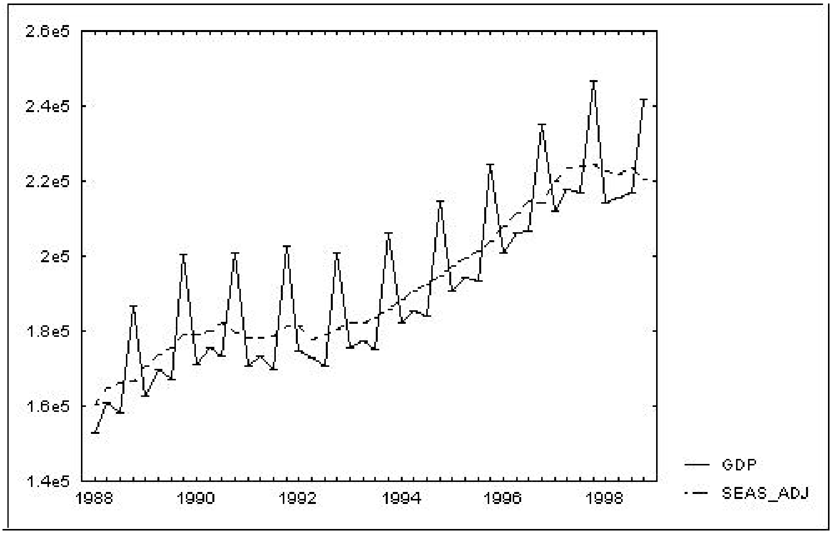
Figure 1. Philippine Gross Domestic ProductFirst Quarter 1981 – Fourth Quarter 1998



Source: Bersales and Sarte, 1999

The following figure, Figure 2, shows the historical plot of the original and seasonally adjusted quarterly GDP from the first quarter of 1988 to the fourth quarter of 1998. Seasonal adjustment was done for the time period 1988 to 1998, 11 years of data, instead of the whole period (1981-1998) for which data are available.

Figure 2. Original and Seasonally Adjusted GDP, 1988 – 1998



Source: Bersales and Sarte, 1999

Many guidelines used in seasonal adjustment resulted in users' experience with various types of data and may actually be considered rules of thumb. The current practice of using computational statistics in developing estimation procedures for complex statistical models may be used to evaluate some guidelines on seasonal adjustment. This paper aims to provide empirical validation of the use of 5 to 15 years of data for seasonal adjustment using simulated data from processes exhibiting the following behavior:

- (Type 1) processes without any structural change
- (Type 2) processes with abrupt permanent change in structure (shift in level)
- (Type 3) processes with gradual changes in structure.

For all three types of processes, realizations with both weak and strong seasonality were simulated.

The following plots illustrate the simulated series:

Figure A.1. Plot of Six Realizations for Type 1 Weak Seasonality

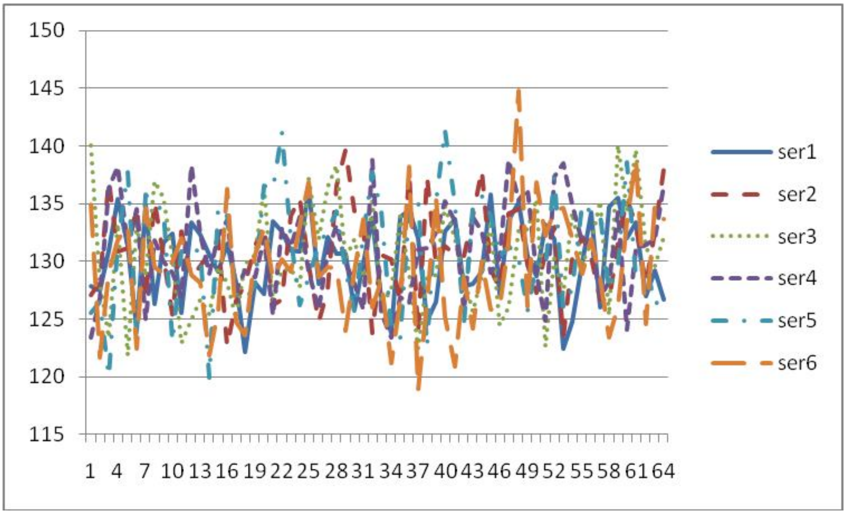


Figure A.2. Plot of Six Realizations for Type 1 Strong Seasonality

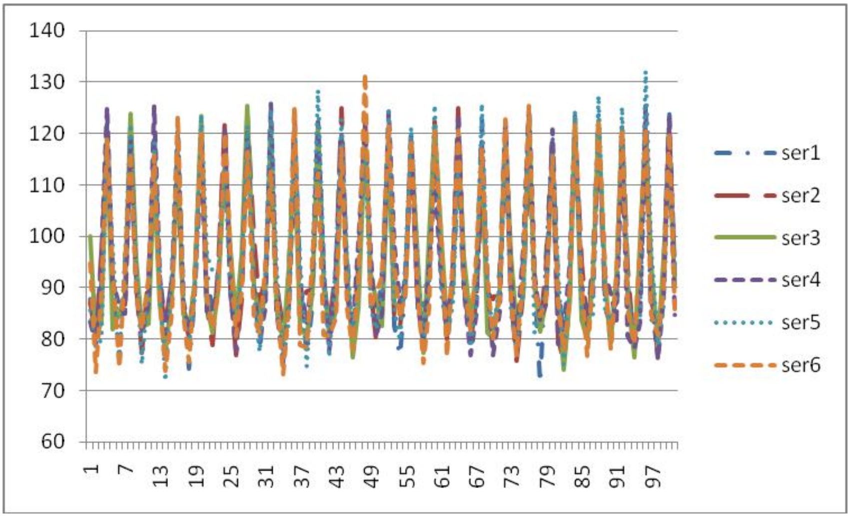


Figure A.3. Plot of Six Realizations for Type 2 Weak Seasonality

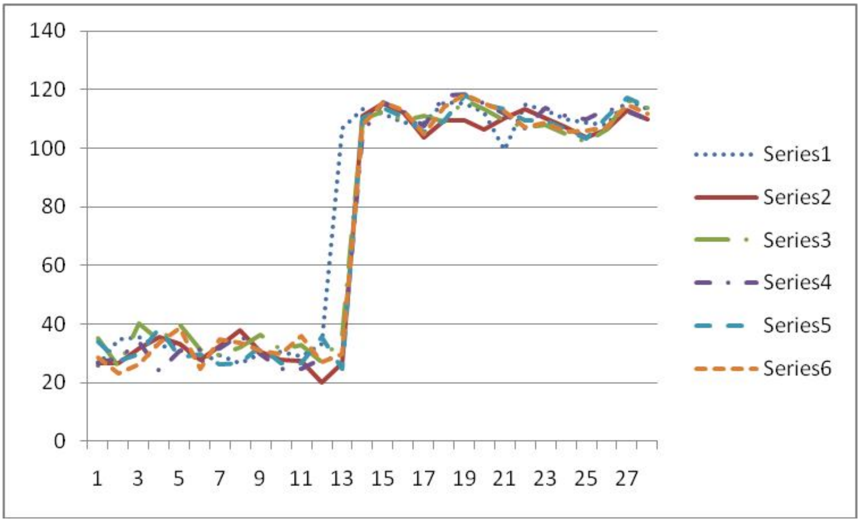


Figure A.4. Plot of Six Realizations for Type 2 Strong Seasonality

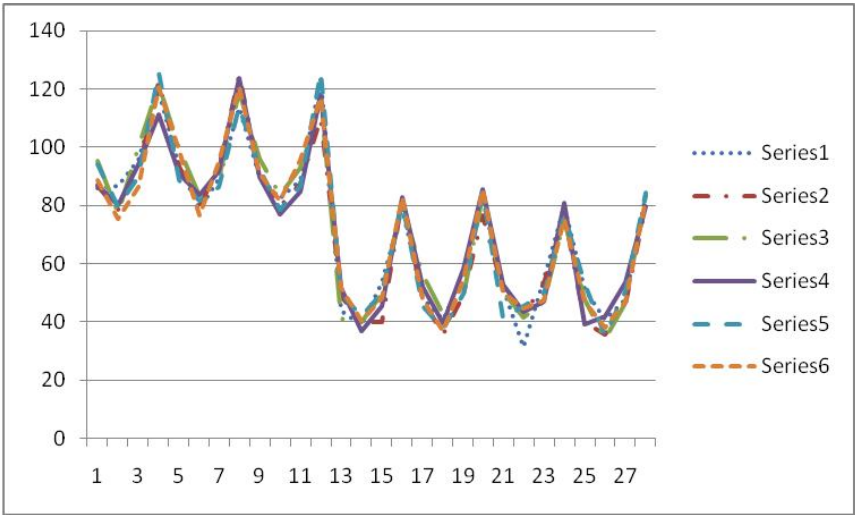


Figure A.5. Plot of Six Realizations for Type 3 Weak Seasonality

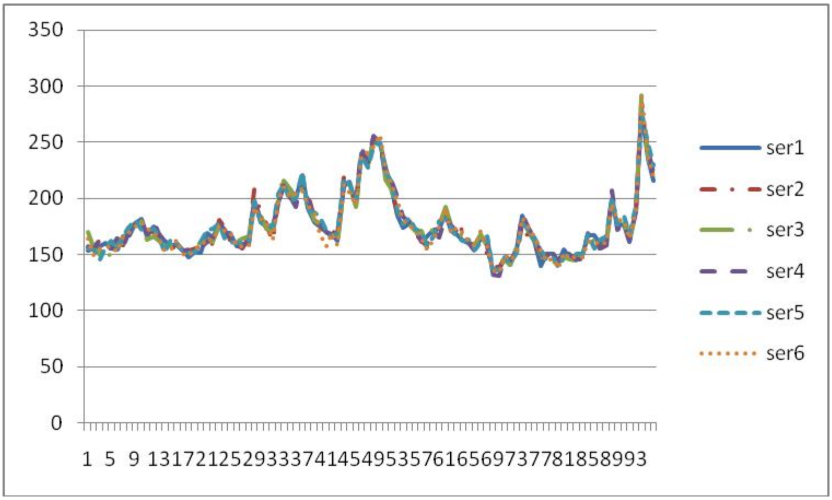
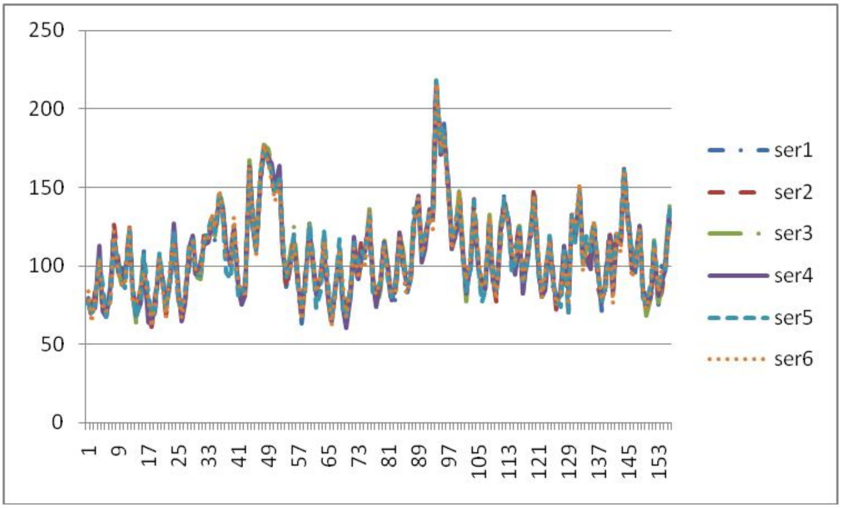


Figure A.6. Plot of Six Realizations for Type 3 Strong Seasonality



2. Methodology

The following procedures were used in achieving the objective of this study:

- *Step 1*.100 realizations with 30 years of quarterly data were generated for each of the following processes which can be modeled as purely Seasonal AR processes:

Type 1

Weak seasonality	Strong seasonality
Y(t)=130 +a(t), a(t)~N(0,4) for Quarter 1	Y(t)=90 +a(t), a(t)~N(0,4) for Quarter 1
Y(t)=128 +a(t), a(t)~N(0,4) for Quarter 2	Y(t)=80 +a(t), a(t)~N(0,4) for Quarter 2
Y(t)=130 +a(t), a(t)~N(0,4) for Quarter 3	Y(t)=90 +a(t), a(t)~N(0,4) for Quarter 3
Y(t)=133 +a(t), a(t)~N(0,4) for Quarter 4	Y(t)=120+a(t), a(t)~N(0,4) for Quarter 4

Type 2

Weak seasonality	Strong seasonality
Pattern 1	*Pattern 1*
Y(t)=30 +a(t), a(t)~N(0,4) for Quarter 1	Y(t)=90 +a(t), a(t)~N(0,4) for Quarter 1
Y(t)=28 +a(t), a(t)~N(0,4) for Quarter 2	Y(t)=80 +a(t), a(t)~N(0,4) for Quarter 2
Y(t)=30 +a(t), a(t)~N(0,4) for Quarter 3	Y(t)=90 +a(t), a(t)~N(0,4) for Quarter 3
Y(t)=33 +a(t), a(t)~N(0,4) for Quarter 4	Y(t)=120+a(t), a(t)~N(0,4) for Quarter 4
Pattern 2	*Pattern 2*
Y(t)=110 +a(t), a(t)~N(0,4) for Quarter 1	Y(t)=50 +a(t), a(t)~N(0,4) for Quarter 1
Y(t)=108 +a(t), a(t)~N(0,4) for Quarter 2	Y(t)=40 +a(t), a(t)~N(0,4) for Quarter 2
Y(t)=110 +a(t), a(t)~N(0,4) for Quarter 3	Y(t)=50 +a(t), a(t)~N(0,4) for Quarter 3
Y(t)=113 +a(t), a(t)~N(0,4) for Quarter 4	Y(t)=80 +a(t), a(t)~N(0,4) for Quarter 4

Type 3

Weak seasonality	Strong seasonality
Weak Type 1 series + GARCH(1.1) errors with ARCH parameter=.239 and GARCH parameter=.667	Strong Type 1 series + GARCH(1.1) errors with ARCH parameter=.239 and GARCH parameter=.667

- *Step 2*.Seasonal adjustment of all realizations were done using the multiplicative decomposition model of X11 with varying lengths of data (5 years to 15 years). For Type 2 processes, the seasonal adjustment concentrated on where the permanent change started (e.g., analysis focused on: 5 years of seasonal adjustment with the first year of data 5 years before the break, 5 years of seasonal adjustment with the first year of data 4 years before the break,…, 5 years of seasonal adjustment with the first year of data is the first year of the new behavior)

- *Step 3.* Seasonal factors of the processes were extracted using the multiplicative decomposition model of X11. The whole length of available data was used in the extraction. For realizations with abrupt permanent change, the seasonal factors for the old pattern and the new pattern were extracted separately. The mean seasonal factors from this step are assumed to be the actual seasonal factors for the process.
- *Step 4.* The mean seasonal factors from Step 2 were compared with the mean seasonal factors from Step 3. This produced the error series which was generated by getting the absolute value of the difference.
- *Step 5.* The following regression models were estimated and tests of significance were done to determine if length of the series used in seasonal adjustment has significant effect on the errors. For series with abrupt break, the regression included an independent variable reflecting first year of seasonal adjustment relative to the break in the series. This determines the quality of will is before or after the new behavior starts.
  - o For processes with abrupt permanent change:

    $E(ERROR) = \theta_0+\theta_1 Q1+\theta_2 Q2+\theta_3 Q3+ \theta_4 LENGTH$ (model 1)
  - o For other processes:

    $E(ERROR) = \alpha_0+ \alpha_1 Q1+\alpha_2 Q2+\alpha_3 Q3+\alpha_4 YEAR +\alpha_5 LENGTH$ (model 2)

  where Q_k is an indicator variable representing quarter k; k=1,2,3

  YEAR= years before/after start of new behavior with value for 0 for the year where new behavior started, -1 year before new behavior, 1 year after new behavior started

  LENGTH= number of years of data used in the seasonal adjustment, with values from 5 to 15.

  Of interest is to test the significance of LENGTH.

Seasonal adjustment and regression analysis were done using Eviews6. Generation of simulated data was done using the following: normally distributed errors for Types 1 and 2 processes in Excel, GARCH (1,1) errors for Type 3 in Eviews6; and, all realizations in Excel.

3. Discussion of Results

Estimation and model 1 and model 2 using weighted least squares in the procedure used to answer the study resulted in Table 1. In the table, AE is ERROR in models 1 and 2.

$E(ERROR) = \theta_0 + \theta_1 Q1 + \theta_2 Q2 + \theta_3 Q3 + \theta_4 LENGTH$ (model 1)

$E(ERROR) = \alpha_0+ \alpha_1 Q1 + \alpha_2 Q2 + \alpha_3 Q3 + \alpha_4 YEAR + \alpha_5 LENGTH$ (model 2)

The results in Table 1 show that the length of series used does not have significant effect on the errors in estimating the seasonal factor for processes without structural change and with abrupt shifts in level. On the other hand, for processes with gradual permanent change, use of longer lengths of series for seasonal adjustment is better.

It is further noted that for processes with abrupt shifts in level, what affects the seasonal adjustment is the start of the data being used for seasonal adjustment. The errors in estimation of the seasonal factor are higher when the start is just before the break starts. Once the seasonal adjustment starts with the new behavior, the estimates have lower errors. This result reinforces the practice of cutting the series for seasonal adjustment once a new pattern starts. Tables 2 and 3 clearly show the improvement in error as YEAR nears 0.

Table 1. Results of Regression of Error in Seasonal Factor Estimate versus Independent Variables

	Independent Variable	Coefficient	Std. Error	t-Statistic	Prob.
	C	0.006928	0.000888	7.802005	0.0000
Type 1	Q1	0.001537	0.000616	2.496280	0.0127
Weak Seasonality	Q2	-0.001412	0.000353	-3.997746	0.0001
	Q3	0.001355	0.000658	2.059321	0.0397
	LENGTH	-8.40E-05	7.35E-05	-1.142788	0.2534
	C	0.233896	0.028815	8.117034	0.0000
Type 1	Q1	-0.237455	0.028012	-8.477048	0.0000
Strong Seasonality	Q2	-0.238818	0.028011	-8.525991	0.0000
	Q3	-0.237864	0.028011	-8.491743	0.0000
	LENGTH	0.000627	0.002300	0.272629	0.7852
	C	0.054570	0.003606	15.13399	0.0000
Type 2	Q1	-0.029148	0.002715	-10.73395	0.0000
Weak Seasonality	Q2	-0.014551	0.002477	-5.873684	0.0000
	Q3	-0.024696	0.002160	-11.43114	0.0000
	YEAR	-0.013260	0.000533	-24.87590	0.0000
	LENGTH	-0.000128	0.000276	-0.461721	0.6444
	C	0.124382	0.007669	16.21886	0.0000
Type 2	Q1	-0.054010	0.004654	-11.60423	0.0000
Strong Seasonality	Q2	-0.042050	0.004673	-8.999352	0.0000
	Q3	-0.074169	0.004240	-17.49398	0.0000
	YEAR	0.021333	0.002457	8.682820	0.0000
	LENGTH	-0.000586	0.000514	-1.141329	0.2540
	C	0.243926	0.037478	6.508445	0.0000
Type 3	Q1	0.055877	0.022488	2.484713	0.0131
Weak Seasonality	Q2	-0.003505	0.020780	-0.168683	0.8661
	Q3	-0.001763	0.020904	-0.084328	0.9328
	LENGTH	-0.013670	0.003135	-4.360837	0.0000
	C	0.152922	0.016125	9.483651	0.0000
Type 3	Q1	0.055870	0.009412	5.935775	0.0000
Strong Seasonality	Q2	-0.007557	0.002514	-3.005926	0.0027
	Q3	-0.009601	0.002557	-3.755392	0.0002
	LENGTH	-0.011323	0.001303	-8.691081	0.0000

Table 2. Descriptive Statistics for AE for Type 1 Weak Seasonality

Categorized by values of YEAR
Sample: 1 1236
Included observations: 1236

YEAR	Mean	Std. Dev.	Obs.
-2	0.058404	0.035671	176
-1	0.072755	0.038330	220
0	0.014147	0.012380	260
1	0.011959	0.010359	296
2	0.010641	0.008583	284
All	0.029552	0.034249	1236

Table 3. Descriptive Statistics for AE for Type 2 Weak Seasonality

Categorized by values of YEARS
Sample: 2 1124
Included observations: 1123

YEARS	Mean	Std. Dev.	Obs.
-2	0.033068	0.037877	36
-1	0.054411	0.052092	380
0	0.028242	0.027535	380
All	0.038657	0.042030	1123

For Type 3 series, Tables 4 and 5 show that it is better to use longer series to seasonally adjust. This result reinforces the current practice of using long length for seasonally adjustment.

Table 4. Descriptive Statistics for AE, Type 3 Weak Seasonality

Categorized by values of LENGTH
Sample: 1 1968
Included observations: 1644

LENGTH	Mean	Std. Dev.	Obs.
5.00	0.362003	0.483999	100
6.00	0.261115	0.442540	112
7.00	0.205172	0.406549	128
8.00	0.168678	0.376878	144
9.00	0.136638	0.345669	156
10.00	0.105658	0.309267	164
11.00	0.048456	0.213433	168
12.00	0.048504	0.213422	168
13.00	0.048539	0.213415	168
14.00	0.048530	0.213417	168
15.00	0.119869	0.324546	168
All	0.126141	0.332062	1644

Table 5. Descriptive Statistics for AE, Type 3 Strong Seasonality

Categorized by values of LENGTH
Sample: 1 1608
Included observations: 1608

LENGTH	Mean	Std. Dev.	Obs.
5.00	0.220717	0.398762	64
6.00	0.110329	0.286633	112
7.00	0.073915	0.230748	128
8.00	0.071291	0.232702	144
9.00	0.070401	0.237083	156
10.00	0.066400	0.231688	164
11.00	0.009104	0.013501	168
12.00	0.008092	0.013655	168
13.00	0.007872	0.013757	168
14.00	0.007569	0.013815	168
15.00	0.007425	0.013868	168
All	0.046525	0.185411	1608

4. Conclusion

This study provides empirical evidence that validates the current practices of official seasonal adjustment of Philippine time series of using long lengths of series when doing seasonal adjustment and cutting the series where a new pattern starts. Results indicate that as long as the series do not have changes in behavior or the change is in an abrupt shift in level, the length of series, as long as within 5 to 15 years, has no significant effect on error of estimation of the seasonal factor. However, for series with more volatile changes, use of longer series is recommended. All these results hold for both weak and strong seasonality.

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